

Math 5SN Midyear Review List

Intro/Review

- Simplifying radicals
- Rationalizing the denominator
- Factoring
- Composition of functions
- Operations on functions
- Linear function – rules & properties
- Quadratic function – rules & properties

Absolute Value Function

- Definition of Absolute Value
- Evaluating a Absolute Value Function
- Graphing
- Given graph find the rule
- Solving equations/finding the zeros
- Comparing functions/applications
- Properties

Square Root Function

- Evaluating a Square Root Function/restrictions
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- Finding the rule given vertex & point
- Inverse of square root function
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- Properties

Rational Function

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- Comparing Functions/Applications
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Optimization

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- Graphing lines/inequalities
- Polygon of constraints
- Finding vertices
- Objective function
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- Basics/Rules
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- Solving equations without logs
- Simple exponential word problems
- Applications of growth & decay
- Applications of compound interest

Logarithmic Function

- Definition
- Switching from log to exp form and vice versa
- Evaluating simple logs
- Basic Theorems
 $\log_b b = 1, \log_b 1 = 0, \log_b b^r = r$
- Theorems - product/addition, quotient/subtraction, exponent/multiplication
- Change of base theorem
- Writing as a single log
- Expanding logs
- Solving log equations: simplify and compare
- Solving log equations: simplify, write in exponential form, solve
- Graphs & properties
- Applications

Name : _____

Math 5 SN Midyear Review

1 A microbiologist is studying two bacteria populations.

Last Monday, the 1st population numbered 2000 and the 2nd numbered 2 048 000.

He noted that the 1st population doubled every day while the 2nd population was reduced by half each day.

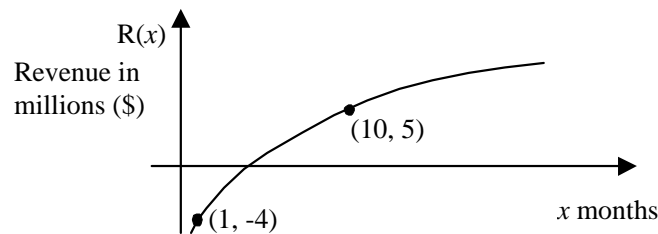
After how many days would the two populations of bacteria be equal in number?

2 At last count, a certain species of bird numbered 200, raising fears that its extinction was imminent. Hunting this species is now forbidden. With this law in place, biologists claim that the population of this species will double every 6 months.

They believe that the species will be saved once its population reaches 18 500.

In how many years will this species no longer be threatened with extinction?

3 After one month in operation, a company's revenue has grown according to the formula $R(x) = a\sqrt{x - h} + k$ where $R(x)$ is the profit in millions of dollars after a period of x months.



Losses amounted to \$4M the first month. After ten months, \$5M in profits was recorded.

After how many months in operation did the company begin to make a profit?

4 Three years ago Greg invested \$1000 at a fixed interest rate compounded every 6 months. His investment is currently valued at \$1400.

Given $C_n = C_0 \left(1 + \frac{t}{k}\right)^{nk}$ where

C_n	is the capital after n years
C_0	is the capital invested
t	is the annual interest rate
k	is the number of times per year that interest is paid
n	is the number of years

To the nearest centimetre, what is the annual rate of interest?

5 The value v , in dollars, of a stock varies with time t , in days. This situation is defined by

$$v(t) = -\left[\frac{t}{2}\right] + 8 \quad \text{if } t \in [0, 6],$$

$$v(t) = \frac{3}{16}(t - 10^2) + 3 \quad \text{if } t \in [6, 10],$$

$$v(t) = 0.1875t + 1.125 \quad \text{if } t \in [10, 20],$$

For how many days was the value v of the stock \$3.75 or less?

6 Solve the following inequation and plot the solution set on the real number line.

$$|2x + 3| \leq 1$$

7 At the beginning of a laboratory experiment, the temperature of a substance is 29°C . Throughout the experiment, the deviation in temperature must follow the formula :

$$|2x - 59| \leq 3$$

where x represents the temperature of the substance.

Between which values can the temperature vary?

8 For a science project, Louise and Bob recorded the temperature outside their school for a 24 hour period. At the beginning, the thermometer read -1°C . The minimum temperature of -5°C was reached 8 hours later.

The data recorded show that the temperature $t(x)$ varied as a function of the number of hours x elapsed and that t is an absolute value function.

During how many hours was the temperature less than or equal to -3°C ?

9 Computers have changed a great deal since 1950 because of the miniaturization of the circuits.

Year	Number of circuits on a chip $f(x)$
1950	35
1960	3 500
1970	350 000
1980	35 000 000
1990	...

The function representing this situation is given by

$$f(x) = 35(10^{wx})$$

where x represents the number of years since 1950 and w , a parameter.

What is the value of the parameter?

- 10 A basket of groceries today costs \$200. If the rate of inflation remains at 4 % for the next few years, how much will the same grocery basket cost in 5 years?

Note : Express your answer to the nearest hundredth.

- 11 A radioactive substance disintegrates at a rate such that after 2 years it has $\frac{4}{9}$ of its initial mass. If you have 60 grams of this substance, how much of it will remain after 12 years?

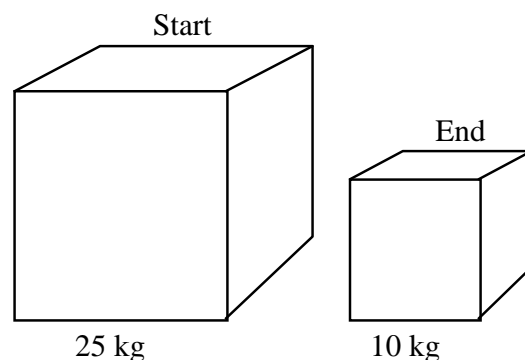
Note : Express your answer in grams to the nearest hundredth.

- 12 On their last fishing trip, Louis and Carmen took a block of dry ice with them.

When they started out, the block had a mass of 25 kg. During the trip, the block sublimated at a constant rate such that every 12 hours its mass decreased by 9 %.

At the end of the trip, the block had a mass of 10 kg.

To the nearest tenth of an hour, how long did their trip last?



- 13 In 1990, the deer population on Anticosti Island was estimated at 70 000. If this population doubles every 50 years, this situation can be translated by the following function:

$$P(t) = (70\,000)2^{\frac{t}{50}}$$

In what year will there be 100 000 deer?

- 14 A car loses 15 % of its value per year for the first 4 years and 10 % a year for the years that follow.

After how many years will a \$28 000 car be worth \$6 632.13?

- 15 A "Super Ball" rebounds to $\frac{3}{4}$ of its previous height after each bounce.

If you drop the "Super Ball" from a height of 20 m, after how many bounces will it reach a height of 2 m?

- 16 During a fireworks show, two rockets must be launched from two different spots at the same time.

The heights, in metres, of the two rockets as a function of time t , in seconds, are represented by the following equations:

$$h_1(t) = -12.5(t - 4)^2 + 200$$

$$h_2(t) = 25\sqrt{\frac{t}{d}} + 50$$

Both rockets must explode 6 seconds after they have been launched. They must also explode at the same height.

What is the value of parameter d ?

- 17 One very hot summer day, the outside temperature is described by the following rule:

$$T(x) = -3|x - 6| + 36$$

where x represents the number of hours that have elapsed since sunrise
 $T(x)$ represents the temperature in $^{\circ}\text{C}$

To make the temperature inside a shopping centre more comfortable, an air conditioning system starts up when the outside temperature reaches 21°C and stops when the outside temperature drops to 20°C .

How long was the air conditioning system in operation on that day?

- 18 The value of a certain amount of capital C_0 invested at a given rate of interest i for t years will be as follows:

$$C(t) = C_0(1 + i)^t$$

Emily invested the \$2000 she received for winning a design contest. This money will earn the same rate of interest throughout the term of the investment.

The following table of values shows the value of Emily's investment as a function of time in years.

Term of the Investment (years)	Value of the Investment (\$)
1	2200
3	2662

How much will Emily's investment be worth after 10 years?

- 19 What is the solution set of the following inequality within \mathfrak{R} ?

$$4 - |3x - 5| > 0$$

Show all your work.

- 20 Jenny and Eric analyzed the changes in the value of Future Telecom's shares in 1999. On January 1st 1999, the initial value of a share was \$25. On May 31st, the share reached its minimum value of \$10. Since then, the value of shares has been on the increase.

They noticed that the relation between the elapsed time, in months, and the value of a share, in dollars, was an absolute value function.

What was the value of a share on December 31st, 1999?

- 21 Reproduction of a certain type of insect is the focus of a laboratory experiment. There were 25 insects at the beginning of the experiment. It was noted that the number of insects increases by 3% every 7 days.

After how many days will there be 20 425 insects?

- 22 The number of people living in Kilwat, Germany, varies according to the rule of an exponential function. On January 1st 1975, the city's population was 130 000. On January 1st 1985, it was 260 000.

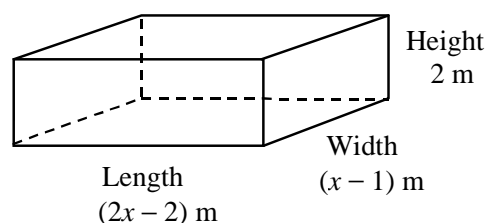
What was the population of this German city on January 1st 2000, given that the growth rate remained constant?

- 23 The towns of Geometrix and Matrix are matched for a cultural exchange. The population of Geometrix is 40 000, while that of Matrix is 10 000. For the next 30 years, experts predict that the population of Geometrix will diminish by 3% per year. During the same period, they expect that the population of Matrix will increase by 5% annually.

In how many years will the two towns have the same population?

- 24 The volume of the right prism represented below is less than or equal to 64 m^3 .

The length of the prism is greater than or equal to 4 m.



For what values of x are these two conditions respected?

25 A family of four spent \$200.00 a week on groceries in 2001.

When will the weekly grocery bill for a family of four be \$250.00 if the cost of living increases by 3% per year?

Round your answer to the nearest tenth.

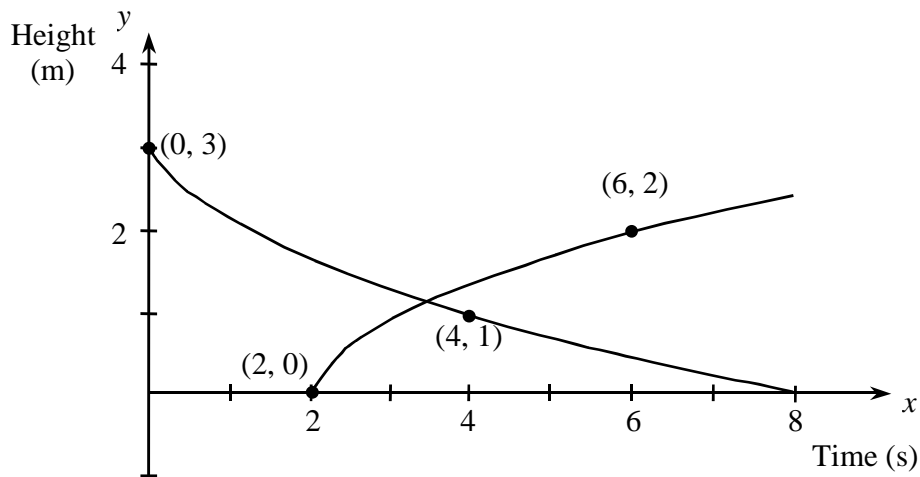
26 Sonia is replacing some of the water in her aquarium without removing the fish.

When she started working, the aquarium contained 60 litres. After 35 seconds, 45 litres of water remained in the aquarium. This is the minimum quantity of water needed to ensure that the fish will not be harmed. Then, using a pump, Sonia filled the aquarium to its maximum capacity of 70 litres.

Sonia noted that the relation between the time elapsed, in seconds, and the quantity of water in the aquarium, in litres, corresponds to an absolute value function.

Throughout this process, for how many seconds was the quantity of water in the aquarium less than 53 litres?

27 Two missiles are launched 2 seconds apart. The paths they follow over a span of 8 seconds can be represented by two different square root functions, as illustrated below:



How many seconds after the 2nd projectile has been launched, will it be higher than the 1st projectile?

28 In a laboratory, the reproduction of a particular species of insect is studied. At the beginning of the experiment, there are 25 insects. The number of insects increases by 30% every 7 days.

After how many weeks will there be 20 425 insects?

29 A car's speed is a function of time, which can be represented by an absolute value function graph.

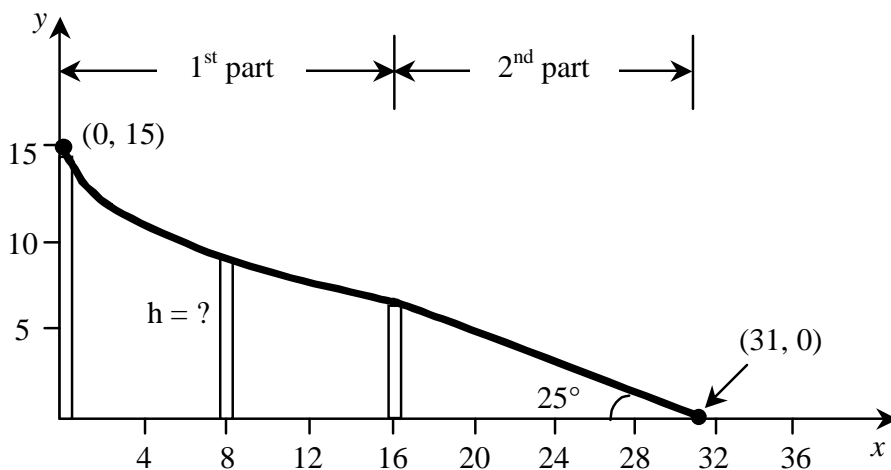
Dean's car goes from rest to a maximum speed of 150 km/h in 12 seconds and slows down at the same rate when he brakes.

For how many seconds is the speed of Dean's car at least 50 km/h but no more than 120 km/h?

30 The graph shown below, with the axes graduated in meters, represents a slide that spans 31 metres in length.

Three posts placed 8 meters apart support the first part of the slide. This part of the slide is represented by a square root function with vertex $(0, 15)$.

The second part of the slide is linear and forms an angle of 25° with the horizontal.



As shown in the graph above, the second post is 8 meters away from the first, which is located on the y -axis.

What is the height of the second post?

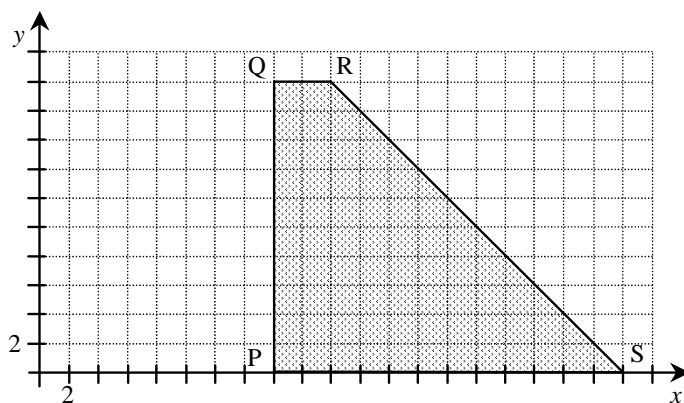
Round your answer to the nearest hundredth of a metre.

- 31 Jonathan works at a golf club during his summer vacation. He sometimes cleans the premises and sometimes works in the kitchen at the club's restaurant. Jonathan makes \$8 per hour when cleaning the premises and \$9.50 per hour when working in the kitchen.

There are certain constraints on the number of hours he can devote to each job every week. This situation is represented by the system of inequalities and the polygon of constraints given below.

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\leq 40 \\ x &\geq 16 \\ y &\leq 20 \end{aligned}$$

x : the number of hours spent cleaning the premises
 y : the number of hours spent working in the kitchen



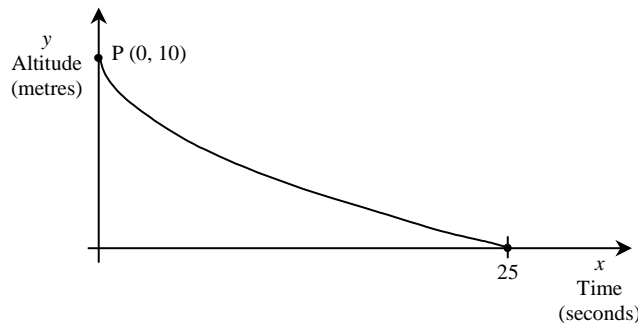
Coordinates of the vertices of the polygon
P(16, 0)
Q(16, 20)
R(20, 20)
S(40, 0)

This week, Jonathan's employer informed him that there would be an additional constraint. This new constraint is represented by the following inequality:

$$x \geq y + 20$$

With this new constraint, by how much will Jonathan's maximum possible income decrease?

- 32 Caroline slides down a waterslide at an aquatic park. The following graph represents Caroline's altitude in relation to her sliding time. This curve represents a square root function whose vertex is point P(0, 10).



A section of the slide is covered by a tarpaulin, forming a tunnel. Caroline enters the tunnel when she is at an altitude of 6 m. She exits the tunnel at an altitude of 4 m.

How long was Caroline in the tunnel?

- 33 When interest is paid n times a year, the value of a certain amount of capital C_0 invested at an annual interest rate i for t years will be as follows:

$$C(t) = C_0 \left(1 + \frac{i}{n} \right)^{nt}$$

Gerry wants to invest \$2 000 for 2 years. He has two investment options.

Investment option A

Annual interest rate of 5%
Interest paid once a year

Investment option B

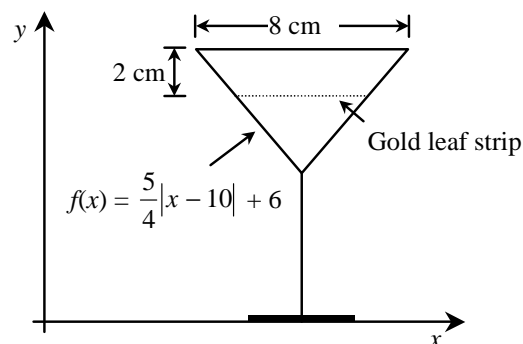
Annual interest rate of 4.2%
Interest paid 12 times a year

Gerry chose investment option A because he was told it would provide the best return.

Rounded to the nearest whole month, how many months would Gerry have had to invest his money under investment option B in order to earn the same amount he will earn under investment option A?

- 34 A glassblower wants to apply a gold leaf strip around a cocktail glass. The gold leaf strip will be applied 2 cm from the rim of the glass.

The side view of this glass is represented in the following Cartesian plane. The scale of the graph is in centimetres.



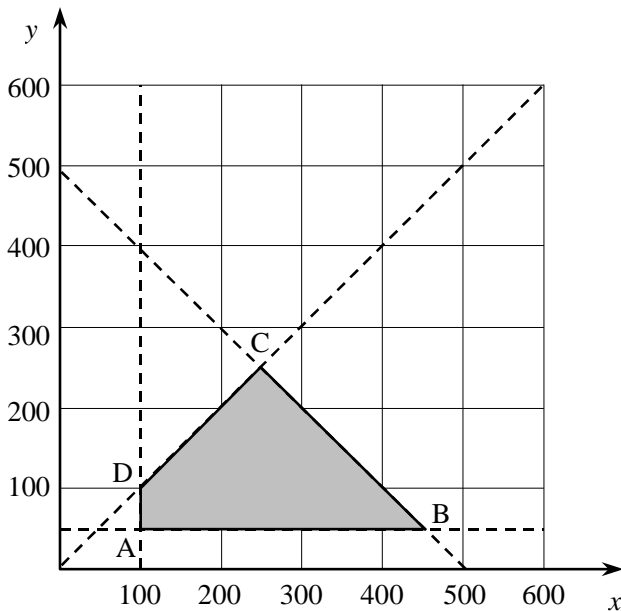
The rule $f(x) = \frac{5}{4}|x - 10| + 6$ is associated with the top part of the glass.

The maximum diameter of the glass is 8 cm.

What is the diameter of the glass at the point where the gold leaf strip will be applied?

35 To raise money, the Graduation Committee decides to sell cases of fruit. The following polygon represents the constraints that must be respected.

If x represents the number of cases of oranges for sale and y , the number of cases of grapefruit for sale, the constraints are:



- ① $x \leq y$
- ② $x + y \leq 500$
- ③ $x \geq 100$
- ④ $y \geq 50$

For each case of oranges and grapefruit sold, the Graduation Committee makes a profit of \$1.00 and \$1.50, respectively.

Yesterday, the head of the committee received a call from the supplier. Because of a recent flood, the supplier can deliver a maximum of 400 cases of fruit.

By how much will the maximum possible revenue decrease because of the flood?

36 Starting 26 weeks before an election, a firm holds a weekly poll on voter intention.

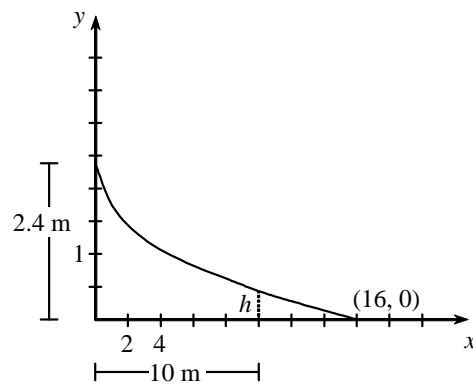
During this polling period, the popularity of Party "A" varies according to an absolute value function.

In the first survey, Party "A" polled 28%. Ten weeks later, the party reached its maximum survey result of 43%.

When did Party "A" have 25% of the survey results?

- 37 Carol's father made a slide during the winter. Placed on a Cartesian plane scaled in metres, the slide follows a square root function, as shown below. The top of the slide, 2.4 m above the ground, coincides with the vertex of the square root function. The foot of the slide is 16 m horizontally from the top.

Carol starts going down the slide. However, her scarf becomes jammed in the sled causing her to stop at a horizontal distance of 10 m.



At what height did she stop?

- 38 An airplane is flying at an altitude of 10 000 m. At 21:00, the pilot begins the descent towards Pierre Elliott Trudeau Airport. The descent follows an exponential model ending with the plane's landing. At 21:04, the airplane is at an altitude of 5222 m.

At what time will the airplane be at an altitude of 280 m?

- 39 The city council of a town wants to minimize the cost of staffing its recreation centres in the summer months. The council has determined that supervisors will be paid \$3500 for the summer and staff workers will be paid \$1500 for the summer.

The council wants to hire its employees using the following constraints:

- The maximum number of employees for its centres is 30 and the minimum is 18.
- The council also wants to hire at least 6 supervisors but no more than 14 supervisors.
- It wants to hire at least 8 staff workers.
- The number of staff workers must be at most twice the number of supervisors.

How many staff workers and how many supervisors can the town council hire and minimize its costs?

- 40 When Jennifer bought a new car in 1995, she paid \$17 500. In 1998 the value of her car had fallen to \$10 000. She decided that she would sell her car when the value fell below \$5000.

Assuming the decline in the price of a car is modelled by an exponential function, how old will Jennifer's car be when its value falls below \$5000? Round your answer to the nearest month.

- 41 Wheeler is a producer of mountain bikes and road bikes. Because of its small size, it can build no more than 80 bikes each week. To meet certain conditions in its workshop, it must build at least 45 mountain bikes, and at least 10 road bikes weekly. To meet consumer demand, it must manufacture at least 3 times as many mountain bikes as road bikes.

The following is the system of constraints for Wheeler's weekly bike production:

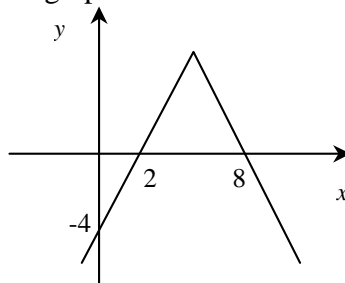
x = the number of road bikes produced weekly
 y = the number of mountain bikes produced weekly

$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x &\geq 10 \\ y &\geq 45 \\ x + y &\leq 80 \\ y &\geq 3x \end{aligned}$$

For each road bike and mountain bike produced, Wheeler earns a profit of \$250 and \$175, respectively. What is the maximum weekly profit that can be earned?

- 42 During one month this spring the value of a share in a travel company behaved according to an absolute value function.

Its initial value was \$4 below its average price.
 On Day 2 and Day 8 it was exactly the same as its average price.
 This situation can be represented by the graph below.



What is the rule of correspondence that represents this situation?

- 43 Each morning, the traffic on a certain highway increases, reaches a peak, and then decreases again.

This situation is represented mathematically by the function:

$$V(t) = -25 | t - 8 | + 65$$

$V(t)$ = the number of vehicles passing per minute
 t = the time, in hours, since midnight

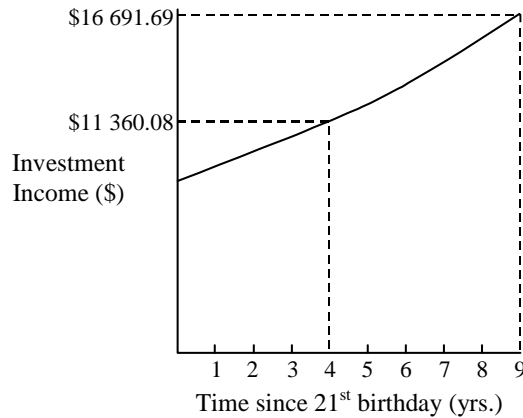
To ensure a smooth and safe flow of traffic, a police officer has been assigned to monitor this section of highway when the volume of traffic is at least 35 vehicles per minute.

For how many hours should the police officer be on duty to ensure the safe flow of traffic?

44 On her 21st birthday, Marie received a lump sum of money. At that time, she decided to invest it at a fixed annual interest rate, compounded yearly, until her 30th birthday.

On her 25th birthday, her investment had grown exponentially to \$11 360.08. On her 30th birthday, it had further grown to \$16 691.69.

This situation is represented in the graph below.



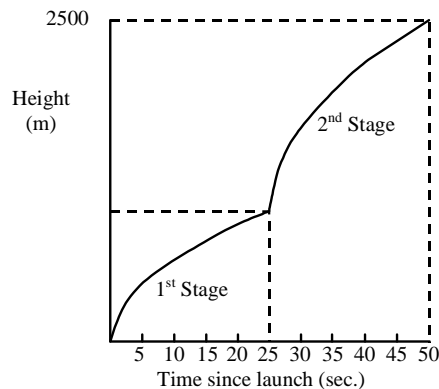
Rounded to the nearest tenth of a percent, what was the fixed annual interest rate over this nine-year period?

45 A two-stage model rocket is launched from ground level. Its 1st stage (engine) powers the rocket vertically, according to the rule $H(t) = 200\sqrt{t}$,

where $H(t)$ is height, in metres, and t is time, in seconds, after launch.

At 25 seconds, the exhausted 1st stage is ejected, and the 2nd stage fires. The height of the rocket after the first 25 seconds can be expressed according to a new square root function of the form $y = a\sqrt{(x - h)} + k$.

50 seconds after the initial launch the rocket reaches a height of 2500 m.



Rounded to the nearest metre, what is the height of the rocket 11 seconds after the firing of the 2nd stage?

- 46 A fisherman has to separate his daily catch of shellfish into two categories before he can sell them. Lobsters are sold for \$8.70 each and crabs are sold for \$9.60 each.

On an average day, the fisherman can expect to catch a minimum of 35 crabs and a maximum of 60. By experience, there are at most twice as many lobsters as crabs in a daily catch and never has the fisherman caught more than 140 shellfish in a single day.

Using a polygon of constraints, determine the maximum revenue that this fisherman can expect to make.

- 47 A virus appeared in South America in the middle of the last decade. Scientists knew that the number of people infected with this virus would increase according to a specific exponential function.

At the beginning of 1996, authorities found 110 infected people. Five years later, the number had grown to 835. Wide-scale inoculation began once 2000 people had been infected with the virus.

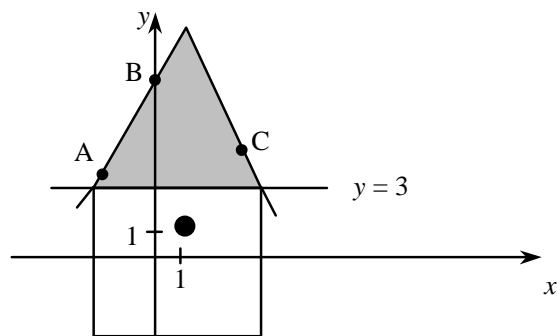
In what year did these inoculations begin?

- 48 In January 1990, there were 5.5 billion people living on this planet. The population has been growing at a rate of 1.9% per year.

In which year will the population reach 9 billion?

- 49 Ethan's diagram, not drawn to scale, shows the front of a birdhouse. The base of the roof corresponds to the line $y = 3$.

The sides of the roof form an absolute value function that passes through the points $A(-2, 4)$, $B(0, 7)$ and $C(4, 5)$.



What is the area of the shaded triangular section of the front of the birdhouse?

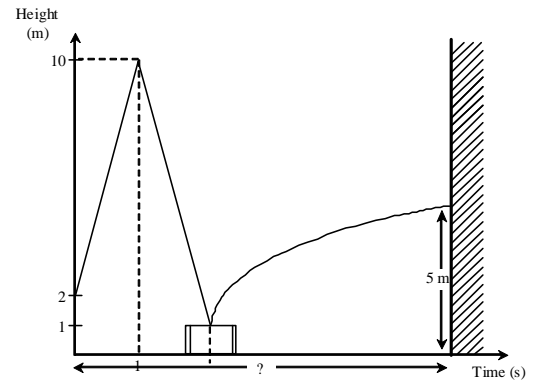
- 50 The Grad Committee plans to sell chocolate bars to raise money for its upcoming dance. This year the committee members have decided to sell two types, one with roasted almonds and the other with caramel. They have a maximum of 500 bars to sell. They expect to sell a minimum of 120 almond chocolate bars. From past experience, almond chocolate bars sell at most 4 times as well as caramel ones. They make a profit of \$0.80 for each almond chocolate bar and \$1 for each caramel chocolate bar.

Let x : number of almond chocolate bars
 y : number of caramel chocolate bars

What is the difference in the maximum profit if they had expected to sell a minimum of 160 almond chocolate bars rather than 120?

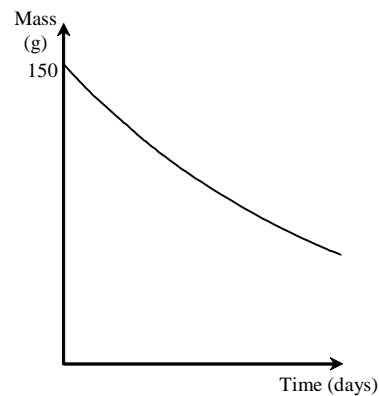
- 51 A tennis ball is hit by a racket from a height of 2 metres and follows the path of an absolute value function. One second later the ball hits the ceiling, which has a height of 10 metres. On its way down, the ball bounces off a table that is 1 metre high. After the bounce, its path is a semi parabola. One second after the ball hits the table, it reaches a height of 3 metres before hitting a wall at a height of 5 metres.

How many seconds after the ball was hit by the racket did it hit the wall?



- 52 A chemist is working with a dangerous compound she has just created. She began with 150 g of the compound, but noticed that it decays exponentially. After observing for 10 days, 123 g remained. She needs to know how long it will take until only half of the compound will be left.

Rounded to the nearest day, how many days after the experiment started will only half of the compound remains?



- 53 Murray plans a trip to New York in July. In order to save money, he works at two different part-time jobs on weekends. At the first job, he works a minimum of 10 hours per month and at the second, a maximum of 40 hours per month. Murray must work at least 30 hours per month but no more than 60 hours per month. He must work at least as many hours at the second job as he does at the first. He makes \$6.30 an hour at the first job and \$8 an hour at the second job.

Let x : number of hours per month at first job
 y : number of hours per month at second job

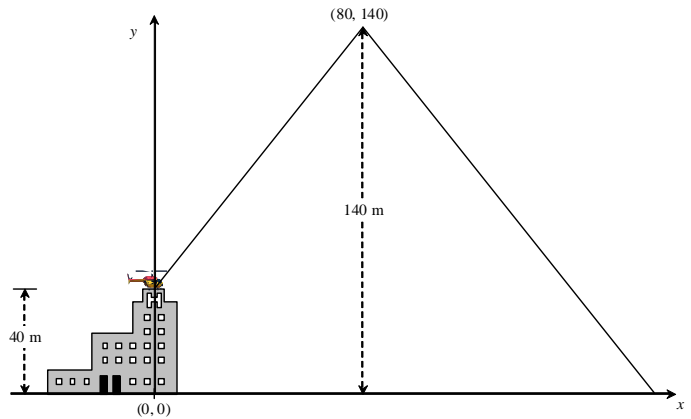
The initial constraints for this situation are:

$$\begin{aligned} x &\geq 10 \\ y &\leq 40 \\ y &\geq 0 \\ x + y &\geq 30 \\ x + y &\leq 60 \\ y &\geq x \end{aligned}$$

Because of a shortage of employees, Murray was later advised that he could increase the number of hours he worked at the second job.

By how much did Murray's maximum possible salary increase because of the employee shortage?

- 54 During an emergency flight, a helicopter left the roof of a 40-metre tall hospital. The helicopter flew at a constant speed and reached a maximum height of 140 meters after 80 seconds. Then, the helicopter descended to the ground at the same speed as it had ascended. The helicopter's flight can be represented by an absolute value function.



After how many seconds was the helicopter at an altitude of 60 m during its ascent and descent?

- 55 In 1991, Albert invested \$4000. In 1999, his investment had grown to \$5474.28. He eventually was able to triple his initial investment.

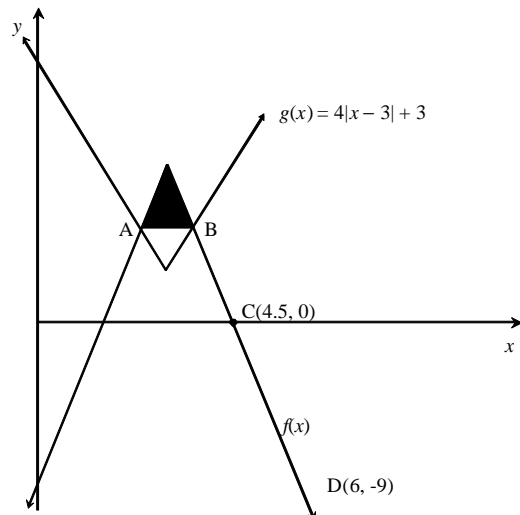
Jocelyn, Albert's brother, also invested a sum of money at the same interest rate. In the number of years it took Albert to triple his initial investment, Jocelyn's investment grew to \$15 000.

What was the difference between Albert's initial investment and Jocelyn's initial investment? (Round your final answer to the nearest dollar.)

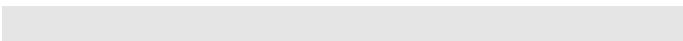
- 56 The logo below was created using two absolute value functions represented by $f(x)$ and $g(x)$. The logo has an axis of symmetry passing through the vertices of $f(x)$ and $g(x)$.

$$g(x) = 4|x - 3| + 3$$

Points A and B represent the points of intersection between $f(x)$ and $g(x)$. Point C(4.5, 0) is one of the zeros of $f(x)$ and $f(x)$ passes through point D(6, -9).



What is the area of the shaded triangular region in the logo?



1 Given the equation : $\log_4(x^2 + 15x) = 2$, where $x \in \mathfrak{R}$.

What value(s) of x satisfies(satisfy) this equation?

2 Solve the following equation :

$$3 \log_a 2 + \log_a(x - 2) = \log_a 12(x + 3) - \log_a 3$$

3 Solve for x in the following equation:

$$\log_3(x + 4) = \log_7 7 - \log_3(x + 2)$$

4 Solve the following logarithmic equation:

$$\log(x + 3) + \log(2x - 7) = \log(2x - 1)$$

5 Solve the following logarithmic equation :

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

6 Solve the following logarithmic equation :

$$\log_2(x - 3) + \log_2(2x) = 3$$

7 Two rival companies A and B decided to make the same product using two different processes.

The following functions represent the gross profits of the two companies:

$$a(t) = 1000 \log_4 t \text{ for company A}$$

$$b(t) = 1000 \log_5 t \text{ for company B}$$

where t is time in months.

After the twelfth month, the expenses of company A were \$800 and company B, \$500.

Which company had the greatest net profit after the twelfth month?

N.B. net profit = gross profit – expenses

8 What is the solution of the following equation?

$$\log_5(x - 1) + \log_5(x + 3) - 1 = 0$$

Math 5SN – Midyear Review Answers

- 1 Result : The two populations will be equal in number after 5 days.
- 2 Answer : This species of birds will no longer be threatened with extinction in about 3.3 years.
- 3 Answer : The company began to make a profit after 2.8 months.
- 4 Answer : The annual rate of interest is 11.52 %.
- 5 Result : The value v of the stock was \$3.75 or less for 6 days.
- 6 $-2 \leq x \leq -1$
- 7 Result : $28 \leq x \leq 31$
- 8 Result : The length of time the temperature was less than or equal to -3 °C was 8 hours.
- 9 Result : The parameter is 0.2.
- 10 Result : \$243.33
- 11 Result : 0.46 grams
- 12 Result : The duration of the trip was 116.6 hours.
- 13 Result : The number of deer will be 100 000 in the year 2015.
- 14 Result : A \$28 000 car will be worth \$6 632.13 after 11.5 years.
- 15 Answer The ball will be 2 m high after 8 bounces.
- 16 Answer The value of parameter d is $\frac{3}{8}$.
- 17 Answer On that day, the air conditioning system was in operation for 10 hours and 20 minutes.
- 18 Answer Emily's investment will be worth \$5187.48 after 10 years.
- 19 Answer The solution set is $\left] \frac{1}{3}, 3 \right[$.
- 20 Answer On December 31st 1999, the share's value was \$31.
- 21 Answer 20 425 insects will be present after 1588 days.
- 22 Answer On January 1st 2000, there were 735 391 people in Kilwat.

- 23 Answer: The two towns will have the same population in 17.49 years.
- 24 Answer: The two conditions are respected for the following values of x : [3, 5] metres.
- 25 Answer: In 7.5 years, the cost of weekly groceries for a family of four will be \$250.
- 26 Answer: The quantity of water in the aquarium is less than 53 litres for 37.3 seconds.
- 27 Answer: 1.36 seconds after it has been launched, the 2nd projectile will be higher than the 1st projectile.

- 28 Answer: There will be 20 425 insects after **179** days.
- 29 Answer: The car's speed will be at least 50 km/h but no more than 120 km/h for 11.2 seconds.
- 30 Answer: The height of the second post is 9.34 meters.

31 Maximum possible income before the new constraint

Vertex	Income: $8x + 9.50y$
P (16, 0)	$8(16) + 9.50(0) = \$128$
Q (16, 20)	$8(16) + 9.50(20) = \$318$
R (20, 20)	$8(20) + 9.50(20) = \$350 \leftarrow$ maximum income
S (40, 0)	$8(40) + 9.50(0) = \$320$

Maximum possible income with the new constraint

Vertex	Income: $8x + 9.50y$
(20, 0)	$8(20) + 9.50(0) = \$160$
(30, 10)	$8(30) + 9.50(10) = \$335 \leftarrow$ maximum income
S (40, 0)	$8(40) + 9.50(0) = \$320$

Difference between the two maximum possible incomes

$$\$350 - \$335 = \$15$$

Answer: With this new constraint, Jonathan's maximum possible income will decrease by **\$15**.

- 32 Answer: Caroline was in the tunnel for **5** seconds.
- 33 Answer: Gerry would have had to invest his money for **28** months, to the nearest whole month, under investment option B in order to earn the same amount he will earn under investment option A.
- 34 Answer: The diameter of the glass at the point where the gold leaf strip will be applied is **4.8** cm.
- 35 Answer: The decrease in revenue caused by the flood is **\$125**.
- 36 Answer: Party "A" has one quarter of the popular vote at **22** weeks.
- 37 Answer: Carol stops at a height of **0.5** m.
- 38 Answer: The airplane will be at an altitude of 280 m at **21:22**.

- 39 Vertices of polygon of constraints:
- | | | | |
|----------|---------------|-------------------------|----------|
| (10, 8) | \Rightarrow | $10(3500) + 8(1500) =$ | \$47 000 |
| (6, 12) | \Rightarrow | $6(3500) + 12(1500) =$ | \$39 000 |
| (10, 20) | \Rightarrow | $10(3500) + 20(1500) =$ | \$65 000 |
| (14, 16) | \Rightarrow | $14(3500) + 16(1500) =$ | \$73 000 |
| (14, 8) | \Rightarrow | $14(3500) + 8(1500) =$ | \$61 000 |

Answer The town should hire 6 supervisors and 12 staff workers in order to minimize its costs.

- 40 Answer The value of the car falls below \$5000 when it is 6.72 years \approx 6 years 9 months.

41	Points (x, y)	Calculation	Profit
	1. (10, 45)	$250(10) + 175(45)$	\$10 375
	2. (15, 45)	$250(15) + 175(45)$	\$11 625
	3. (20, 60)	$250(20) + 175(60)$	\$15 500
	4. (10, 70)	$250(10) + 175(70)$	\$14 750

Answer The maximum weekly profit is \$15 500.

- 42 Answer The rule of correspondence is $y = -2|x - 5| + 6$.

- 43 Answer The police officer should be on duty for 2.4 hours.
(Accept 2 hours and 24 minutes)

- 44 Answer The annual fixed interest rate is approximately 8%.

- 45 Answer Rounded to the nearest metre, the height of the rocket 11 seconds after firing of the 2nd stage was 1995 m.

- 46 Answer: The maximum revenue this fisherman can expect to make is **\$1272**.

- 47 Answer: The population will be offered the vaccine in the year **2003**.

- 48 Answer: The population will reach 9 billion in the year **2016**.

- 49 Answer: The area of the shaded triangular section of the birdhouse is **$24u^2$** .

- 50 Answer: The difference in the maximum profit is **\$8**.

- 51 Answer: The ball hits the wall **6.125** seconds after it was hit by the racket.

- 52 Answer: To the nearest day, half of the compound will remain after **34** days.

- 53 Answer: Murray's maximum possible salary increased by \$17

- 54 Answer: The helicopter was at an altitude of 60 m after **16** seconds as it ascended and **144** seconds as it descended.

- 55 Answer: The difference between Albert's and Jocelyn's initial investments is **\$1002**.

- 56 Answer: The area of the shaded triangular region is **2.16** units².

Answers to Log Practice

1 The values that satisfy this equation is (are) : $x = -16$ and $x = 1$.

2 In this equation, x equals 7.

3 The value of x is -1 .

4 The solution set is $x = 4$

5

$$\log_2(x^2 + 5) - \log_2 5 = \log_2 6$$

$$\log_2 \frac{(x^2 + 5)}{5} = \log_2 6$$

$$\frac{x^2 + 5}{5} = 6$$

$$x^2 + 5 = 30$$

$$x^2 = 25$$

Result : $x = -5$ or $x = 5$

6

$$\log_2(x - 3)(2x) = 3$$

$$2^3 = (x - 3)(2x)$$

$$8 = 2x^2 - 6x$$

$$0 = 2x^2 - 6x - 8$$

$$0 = (2x - 8)(x + 1)$$

$$0 = 2x - 8 \quad \text{or} \quad x + 1 = 0$$

$$x_1 = 4 \quad x_2 = -1 \text{ Value to reject}$$

Answer : x equals 4.

7 Gross profit of company A after the 12th month

$$a(t) = 1000 \log_4 12 = 1000 \frac{\log 12}{\log 4} \approx 1000 \times 1.792 481 \approx 1792.48$$

Gross profit of company B after the 12th month

$$b(t) = 1000 \log_5 12 = 1000 \frac{\log 12}{\log 5} \approx 1000 \times 1.543 959 \approx 1543.96$$

Net profit of company A after the 12th month.

$$\$1792.48 - \$800.00 = \$992.48$$

Net profit of company B after the 12th month.

$$\$1543.96 - \$500.00 = \$1043.96 \quad \text{Result : Company B}$$

8
$\log_5(x - 1) + \log_5(x + 3) - 1 = 0$ $\log_5(x - 1)(x + 3) = 1$ Sum of logs = log of the product $(x - 1)(x + 3) = 5^1$ $x^2 + 2x - 3 = 5$ $x^2 + 2x - 8 = 0$ $(x - 2)(x + 4) = 0$ $x - 2 = 0 \quad \text{or} \quad x + 4 = 0$ $x = 2 \quad \text{or} \quad -4$ -4 is an extraneous root Answer The solution of the equation is 2.