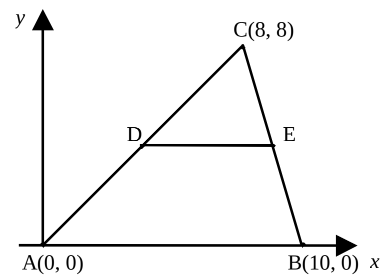
1 Your are asked to prove the following proposition :

The segment joining the midpoints of two sides of a triangle is parallel to the third side and is half as long as the third side.

To prove : in ΔABC
A(0, 0) B(10, 0) C(8, 8)
D is the midpoint of $\overline{AC}$
E is the midpoint of $\overline{BC}$
Conclusion : $\overline{DE} \,//\, \overline{AB}$

$$m\,\overline{DE} = \frac{1}{2}\,m\,\overline{AB}$$



Statements	Justifications
1. The coordinates of point D are (4, 4) and those of point E are (9, 4)	1. ______?______
2. The slope of $\overline{DE}$ is : $\frac{4-4}{9-4} = \frac{0}{5} = 0$	2. Formula for slope
3. The slope of $\overline{AB}$ is : $\frac{0-0}{10-0} = \frac{0}{10} = 0$	3. Formula for slope
4. Therefore $\overline{DE} \,//\, \overline{AB}$	4. ______?______
5. d(D, E) = |9 – 4| = 5	5. Distance formula
6. d(A, B) = |10 – 0| = 10	6. Distance formula
7. Therefore $m\,\overline{DE} = \frac{1}{2}\,m\,\overline{AB}$	7. By substitution of the measures

What are the appropriate justifications to support statements 1 and 4?

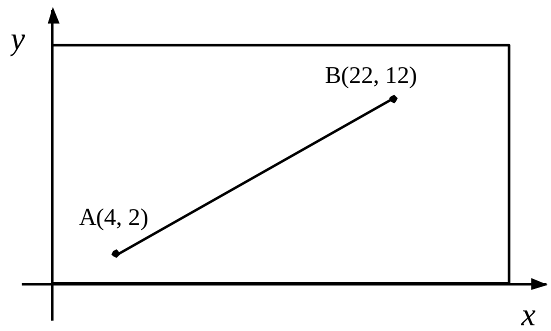
A) 1. Formula for the distance between a point and a line
4. Two lines are parallel if their slopes are equal.

B) 1. Formula for midpoint
4. Two lines are parallel if their slopes are equal.

C) 1. Formula for the distance between a point and a line
4. Two lines, not parallel to the vertical axis, are perpendicular if and only if their slopes are opposite of the reciprocals of each other.

D) 1. Formula for midpoint
4. Two lines, not parallel to the vertical axis, are perpendicular if and only if their slopes are opposite of the reciprocals of each other.

2 Vincent used a Cartesian plane to represent certain objects that make up a theatre set.



A third object must be placed on segment AB, at a point C located $\frac{3}{5}$ of the way along segment AB, starting from point A.

Identify the coordinates of point C.

A) $\left(\frac{74}{5}, 8\right)$

B) $\left(\frac{56}{5}, 6\right)$

C) $\left(\frac{54}{5}, 6\right)$

D) $\left(\frac{31}{4}, \frac{43}{4}\right)$

3 In a Cartesian plane, a line passes through the points A(-10, 6) and B(2, -3). C is a point on segment AB such that the measure of segment AC is equal to $\frac{2}{3}$ of the measure of segment AB $\left(\text{m }\overline{AC} = \frac{2}{3}\text{m }\overline{AB}\right)$.

What are the coordinates of point C?

A) (-6, 3)

B) (-4, 2)

C) (-2, 0)

D) $\left(-\frac{26}{5}, \frac{12}{5}\right)$

4 The coordinates of the vertices of a triangle are (1, 2), (5, 5) and (-2, 6).

In which interval does the area of this triangle fall?

A) [10, 12[

B) [12, 14[

C) [16, 18[

D) [24, 26[

5 The equations of two parallel lines are as follows:

$$2x - 5y - 10 = 0$$
$$2x - 5y + 4 = 0$$

Rounded to the nearest tenth, what is the distance between these two lines?

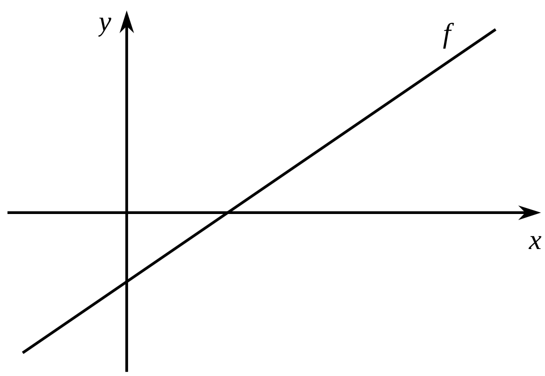
A) 2.5 units

B) 2.6 units

C) 2.7 units

D) 2.8 units

6 A function f is represented in the Cartesian plane below.



Which of the following statements is true?

A) The zero and the y-intercept of function f are positive.

B) The zero and the y-intercept of function f are negative.

C) The zero of function f is negative and the y-intercept of function f is positive.

D) The zero of function f is positive and the y-intercept of function f is negative.

7 Line *l* passes through point P (5, 8) in the Cartesian plane. Line *l* does not have a y-intercept.

What is the equation of line *l*?

A) $x = 5$

B) $x = 8$

C) $y = 5$

D) $y = 8$

8 The equation of line ℓ is $\frac{x}{r} + \frac{y}{t} = 1$, where $r > 0$, $t > 0$ and $r \neq t$.

Which of the following equations represents a line coincident with line ℓ?

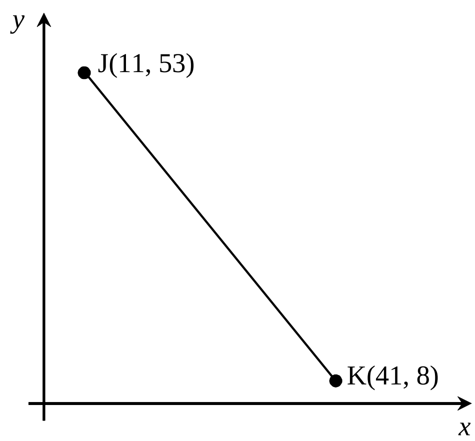
A) $y = -\frac{t}{r}x + r$

B) $y = -\frac{t}{r}x + t$

C) $y = -\frac{r}{t}x + r$

D) $y = -\frac{r}{t}x + t$

9 Point P(29, 26) is a point on line segment JK in the Cartesian plane on the right.



Which of the following statements describes the position of point P?

A) Point P is located $\frac{2}{3}$ of the way along segment JK, starting from point J.

B) Point P is located $\frac{2}{5}$ of the way along segment JK, starting from point J.

C) Point P divides segment JK in a ratio of 3:2, starting from point J.

D) Point P divides segment JK in a ratio of 5:3, starting from point J.

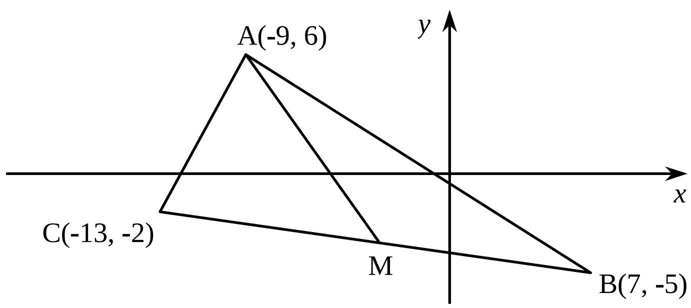
10 Points A, P and B are located on the same line. Point P is located between points A and B. The coordinates of points A and B are (2, -4) and (12, 1) respectively.

In addition, $\dfrac{m\,\overline{AP}}{m\,\overline{AB}} = \dfrac{3}{5}$.

What are the coordinates of point P?

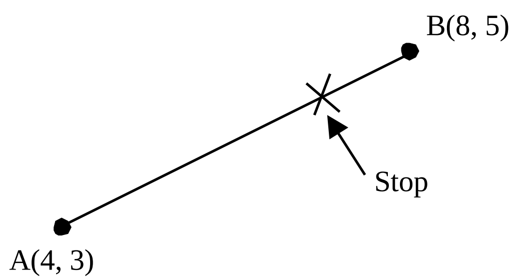
11 Find the coordinates of the mid-point M of segment AB whose end-points are : A(-1, -1) and B(4, 4).

12 In the Cartesian plane below, segment AM is a median of triangle ABC.



What are the coordinates of point M?

13 The following diagram shows a task that has to be done on an automobile driving test.

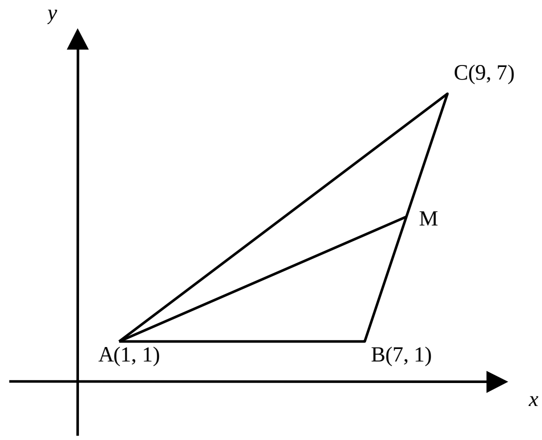


The instructor locates a stop sign $\dfrac{3}{4}$ of the way from A along segment AB.

What are the coordinates of the point that represents the location of the stop sign?

Show all the work needed to solve the problem.

$\boxed{14}$ In a Cartesian plane, the coordinates of the vertices of triangle ABC are A(1, 1), B(7, 1) and C(9, 7). The median AM is also drawn.

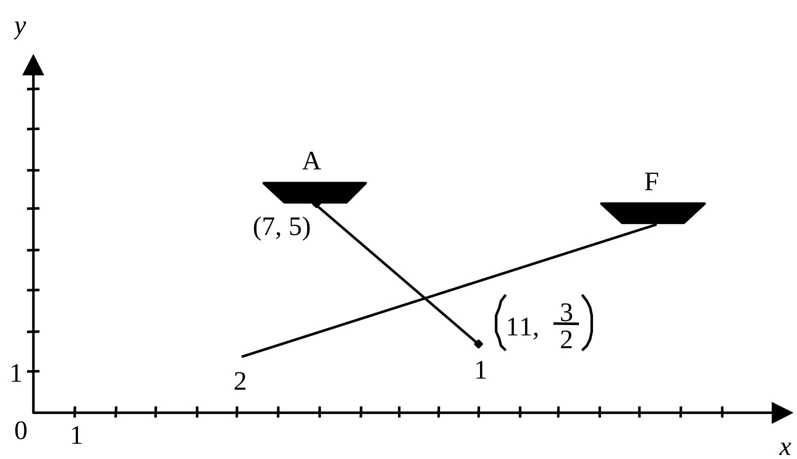


What is the perimeter of triangle AMB?

Show all the work needed to solve the problem.

$\boxed{15}$ Martin (M) was returning to wharf 1, having spent the day fishing. When he was $\frac{2}{3}$ of the way back, he met Jason (J) and they talked about their respective catches.

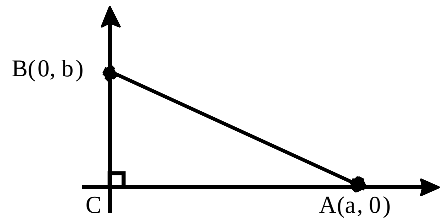
A graphic representation of the situation is shown below.



What are the coordinates of the meeting point of these two fishermen?

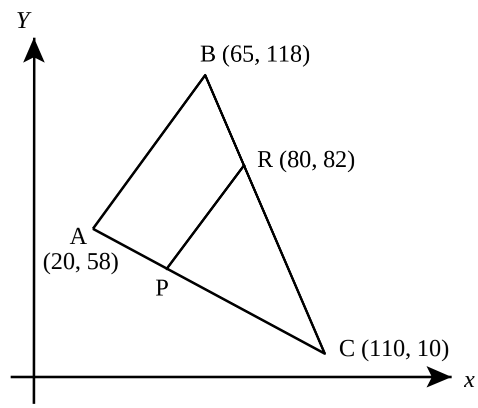
Show your work.

16 Given the right triangle shown on the right.



Prove that the midpoint of the hypotenuse of this right triangle is equidistant from the three vertices.

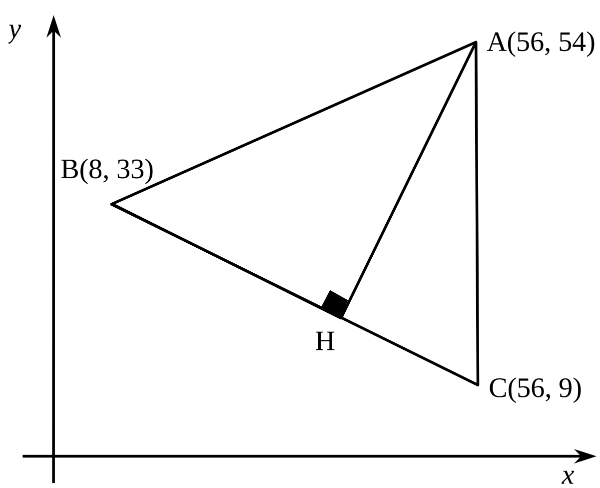
17 In the Cartesian plane on the right, a dilatation with centre C is applied to triangle ABC in order to produce triangle PRC.



What are the coordinates of point P?

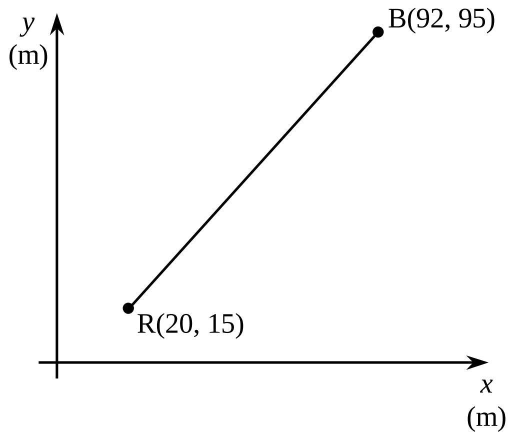
Show all your work.

18 Points A(56, 54), B(8, 33) and C(56, 9) are the vertices of a triangle. Segment AH is an altitude of this triangle.



What is the measure of altitude AH to the nearest tenth?
Show all your work.

19 In a cadet camp, two teams are assigned the task of finding an object hidden along a linear path. When the activity begins, the red team sets off from point R of the path, and the blue team sets off from point B. Path RB is represented in the following Cartesian plane. The scale of the graph is in metres.



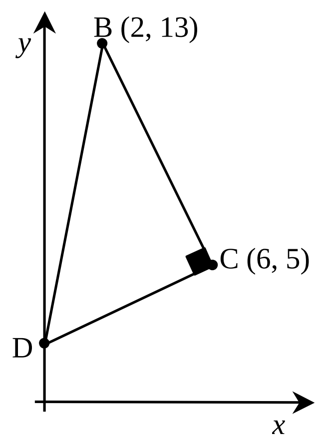
By the end of the activity, the red team's position divides path RB in the ratio 3:5. At the same time, the blue team is located one quarter of the distance from point B to point R.

Rounded to the nearest metre, what is the distance between the two teams by the end of this activity?

Show all your work.

20 In triangle BCD drawn in the Cartesian plane on the right, m∠BCD = 90°.

Point D is located on the *y*-axis.



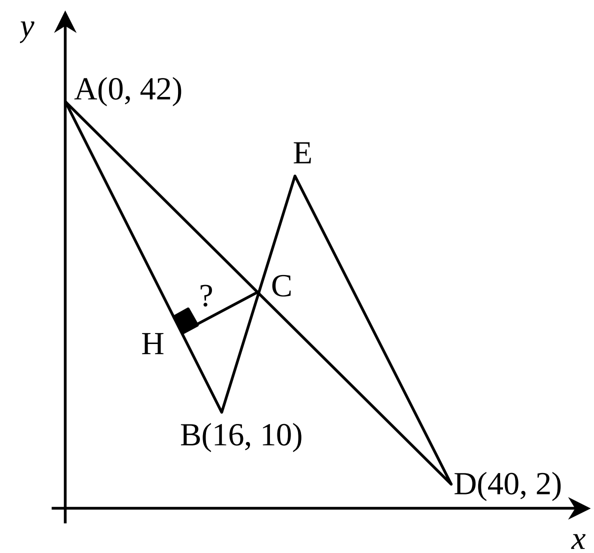
What is the length of hypotenuse BD, to the nearest hundredth?

Show all your work.

21 In the Cartesian plane below:

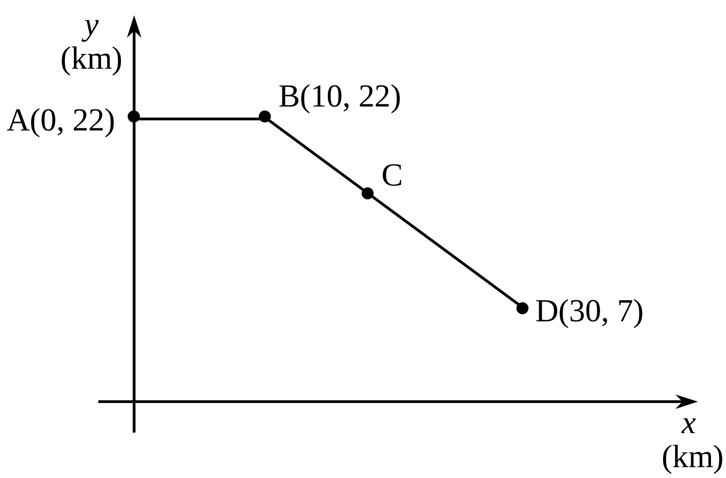
$\overline{AB} \,//\, \overline{ED}$.

Line segments AD and BE intersect at C.
Triangles ABC and DEC are congruent.
Segment CH is an altitude of triangle ABC.



What is the length of altitude CH to the nearest tenth?
Show all your work.

22 In the following Cartesian plane, line segment AB and BD represent two streets along which rally participants must travel. Points A, B, C and D represent checkpoints set up for the rally.



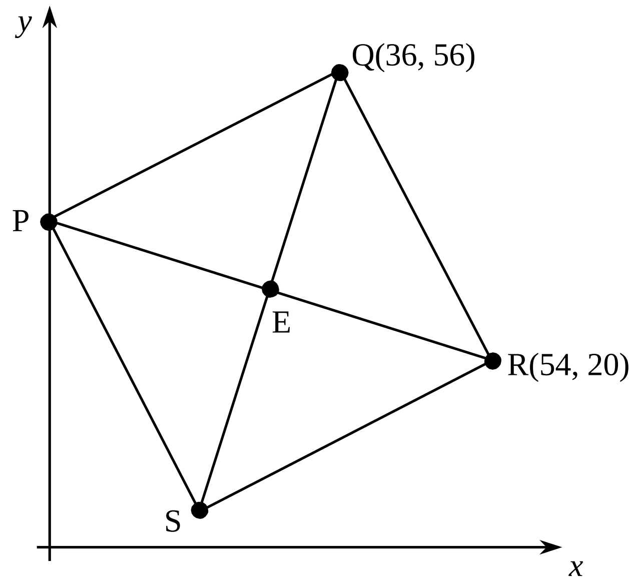
The distance between checkpoints A and B is equal to the distance between checkpoints B and C.
What are the coordinates of point C?

Show all your work.

23 Consider square PQRS in the Cartesian plane below.

Vertex P is located on the *y*-axis.

Diagonals PR and QS intersect at E.

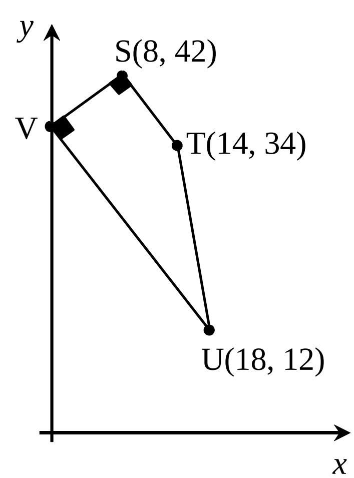


What are the coordinates of point E?
Show all your work.

24 Right trapezoid STUV is represented in the Cartesian plane on the right.

The coordinates of vertices S, T and U are S(8, 42), T(14, 34) and U(18, 12).

Vertex V is located on the *y*-axis.



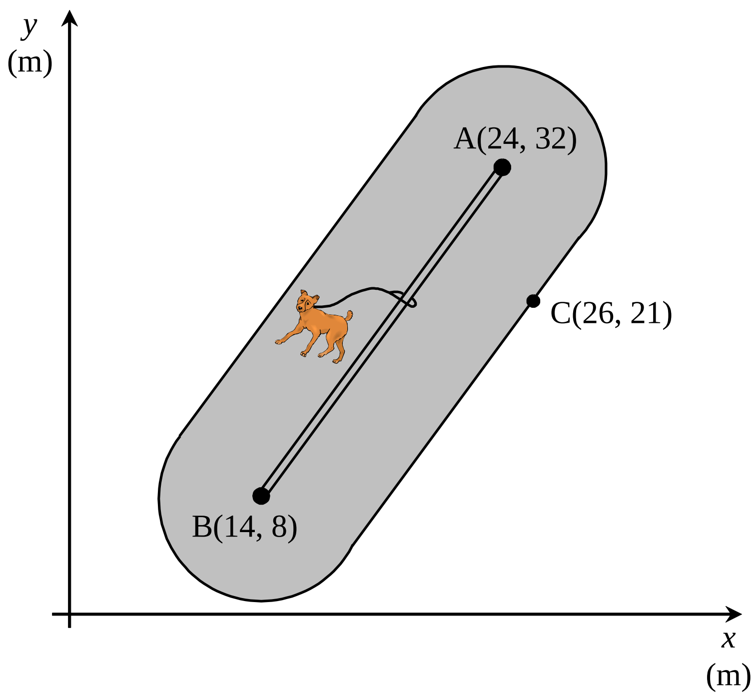
What is the *y*-coordinate of vertex V?

Show all your work.

25 In a park, there are two posts joined by a cable. Points A(24, 32) and B(14, 8) represent the positions of the posts.

A dog's leash is attached to this cable so that he can move freely along the cable. In the Cartesian plane below, the region in which the dog can move about is shaded.

Point C(26, 21) represents a bone located at the edge of this region. The scale of this graph is in metres.



To the nearest square metre, what is the area of the region in which the dog can move about?

Show all your work.

Analytic Geometry Practice

1 B

2 A

3 C

4 B

5 B

6 D

7 A

8 B

9 C

10 The coordinates of point P are: (8, -1)

11 The mid-point is : M(1.5, 1.5)

12 The coordinates of the point are M(-3, -3.5).

13 Endpoints of segment : A(4, 3) and B(8, 5)

Shared point C is located $\frac{3}{4}$ of the way from point A.

Final answer The coordinates of the point that represents the location of the stop sign are (7, 4.5).

14 Coordinate of midpoint M of $\overline{BC}$

M(8, 4)

$$m\,\overline{AM} \approx 7.62$$

$$m\,\overline{MB} \approx 3.16$$

$$m\,\overline{AB} = |7 - 1| = 6$$

Perimeter of ΔAMB

(7.62 + 3.16 + 6) = 16.78

Final answer The perimeter of triangle AMB is 16.78 units.

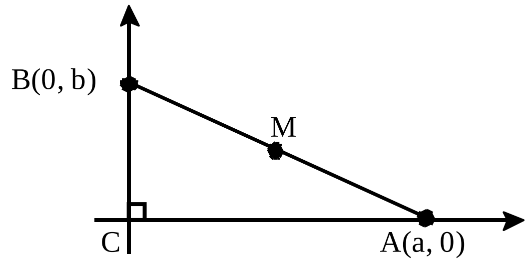
15 Ratio $\frac{2}{3}$

Coordinates of the meeting point :

Result The coordinates of the meeting point are : $\left(9\frac{2}{3}, 2\frac{2}{3}\right)$

16

Let M be the midpoint of the hypotenuse
The coordinates of point M



$$x = \frac{x_2 + x_1}{2} \text{ and } y = \frac{y_2 + y_1}{2}$$

$$x = \frac{a+0}{2} \text{ and } y = \frac{0+b}{2}$$

$$\mathrm{M}\left(\frac{a}{2}, \frac{b}{2}\right)$$

We must prove that d(B, M) = d(A, M) = d(C, M)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d(\mathrm{B, M}) = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - b\right)^2} = \sqrt{\frac{a^2 + b^2}{4}}$$

$$d(\mathrm{A, M}) = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\frac{a^2 + b^2}{4}}$$

$$d(\mathrm{C, M}) = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \sqrt{\frac{a^2 + b^2}{4}}$$

Hence, d(B, M) = d(A, M) = d(C, M)

17

Scale factor

$$\mathrm{m}\ \overline{\mathrm{CR}} = \sqrt{(110 - 80)^2 + (10 - 82)^2} = 78$$

$$\mathrm{m}\ \overline{\mathrm{CB}} = \sqrt{(110 - 65)^2 + (10 - 118)^2} = 117$$

$$\text{Scale factor} = \frac{\mathrm{m}\ \overline{\mathrm{CR}}}{\mathrm{m}\ \overline{\mathrm{CB}}} = \frac{78}{117} = \frac{2}{3}$$

Point P is located $\frac{2}{3}$ of the way along segment CA starting from point C.

Answer The coordinates of point P are (50, 42).

18

$$\text{Slope BC: } \frac{9 - 33}{56 - 8} = \frac{-24}{48} = -0.5$$

y-intercept of BC : 37 = b
Equation of line BC: $y = -0.5x + 37$

$\mathrm{m}\ \overline{\mathrm{AH}}$ = distance between A and the line passing through B and C

$$\mathrm{m}\ \overline{\mathrm{AH}} = \frac{\left|-0.5 \times 56 - 54 + 37\right|}{\sqrt{(-0.5)^2 + 1}} = 40.2492...$$

Answer The measure of altitude AH to the nearest tenth is 40.2 units.

19 The teams never pass each other.

Length of segment RB

$$\sqrt{(92-20)^2 + (95-15)^2} = \sqrt{11\,584} = 107.6289...$$

Distance travelled by the red team

$$\frac{3}{8} \text{ of } m\ \overline{RB} = \frac{3}{8} \text{ of } 107.6289 = 40.3608\ldots$$

Distance travelled by the blue team

$$\frac{1}{4} \text{ of } m\ \overline{RB} = \frac{1}{4} \text{ of } 107.6289 = 26.9072\ldots$$

Distance between the two teams

107.6289 – 40.3608 – 26.9072 = 40.3609

Answer: Rounded to the nearest metre, the distance between the two teams by the end of this activity is 40 m.

The following are the steps associated with another appropriate method:

– Coordinates of the red team's position: (47, 45)
– Coordinates of the blue team's position: (74, 75)
– Distance between their positions: 40.3608…

20 Slope of segment BC $= -2$

Equation associated with segment DC

Slope: $\frac{1}{2}$ since $\overline{BC} \perp \overline{DC}$

2 = b

$y = \frac{1}{2}x + 2$

Coordinates of D

D(0, 2)

Length of hypotenuse BD

$$d(B, D) = \sqrt{(2-0)^2 + (13-2)^2} = \sqrt{125} \approx 11.180$$

Answer: The length of hypotenuse BD, to the nearest hundredth, is **11.18** units.

21 **Coordinates of C**

Since Δ ABC $\cong$ Δ DEC and $\overline{AB} \,/\!/\, \overline{ED}$, $\overline{AC}$ and $\overline{DC}$ are congruent, corresponding segments.

Point C is therefore the midpoint of segment AD. C(20, 22)

Equation associated with segment AB

y-intercept: 42

Slope: $\frac{42-10}{0-16} = \frac{32}{-16} = -2$

$y = -2x + 42$

$2x + y - 42 = 0$

The length of altitude CH is equal to the distance between point C and segment AB.

$$\frac{|2(20) + 1(22) - 42|}{\sqrt{2^2 + 1^2}} = \frac{20}{\sqrt{5}} \approx 8.944$$

Answer: To the nearest tenth, the length of altitude CH is **8.9** units.

22

Length of segment BC

$$m\ \overline{AB} = 10 - 0 = 10$$

$$m\ \overline{BC} = m\ \overline{AB} = 10$$

Length of segment BD

$$m\ \overline{BD} = \sqrt{(30-10)^2 + (7-22)^2} = \sqrt{625} = 25$$

Location of point C

$$\frac{m\ \overline{BC}}{m\ \overline{BD}} = \frac{10}{25} = \frac{2}{5}$$

Answer: The coordinates of point C are C**(18, 16)**.

23

Slope of segment QR $= -2$

Slope of segment PQ: $\frac{1}{2}$

Coordinate of point P

y-intercept of segment PQ

$38 = b$

Coordinates of point P: P(0, 38)

Coordinates of point E

Since the diagonals of a square bisect each other, point E is the midpoint of segment PR.

$E(27, 29)$

Answer: The coordinates of point E are E(**27, 29**).

24

Slope of segment ST $= -\frac{4}{3}$

Segments ST and VU are parallel because they correspond to the bases of the trapezoid. They therefore have the same slope.

Slope of segment VU = $-\frac{4}{3}$

Equation of the line associated with segment VU

$36 = b$

$$y = -\frac{4}{3}x + 36$$

The y-coordinate of vertex V corresponds to the y-intercept of segment VU; the y-coordinate is therefore 36.

y-coordinate of vertex V = 36

Answer: The *y*-coordinate of vertex V is **36**.

25 Example of an appropriate method

Length of segment AB $= 26 \text{ m}$

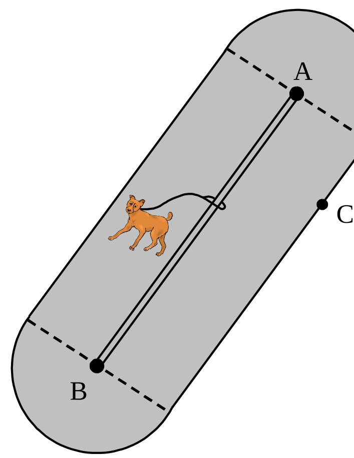
Equation of the line associated with segment AB $12x - 5y - 128 = 0$

Distance from point C to segment AB

$$\frac{|12(26) - 5(21) - 128|}{\sqrt{12^2 + (-5^2)}} = \frac{79}{13}$$

$$\approx 6.077 \text{ m}$$

Area of the region in which the dog can move about



The region can be divided into a rectangle and two congruent semicircles with a radius of 6.077 m.

Area of the rectangle

$26 \times 12.154 \approx 316 \text{ m}^2$

Area of the semicircles

$\pi \times (6.077)^2 \approx 116.02 \text{ m}^2$

Area of the region

$316 + 116.02 \approx 432.02 \text{ m}^2$

Answer: To the nearest square metre, the area of the region in which the dog can move about is **432** m^2.