Math 4CST Name: Key

More practice on Step, Periodic and Piecewise Functions

1. Given the following function:

$$f(x):\begin{cases} = -2x + 50,\ 0 \le x < 10 \\ = 3x,\ 10 \le x < 20 \\ = x + 40,\ 20 \le x \le 100 \end{cases}$$

Evaluate each of the following:

a) $f(3) = -2(3) + 50$
$= 44$

b) $f(10) = 3(10)$
$= 30$

c) $f(19) = 3(19)$
$= 57$

d) $f(20) = 20 + 40$
$= 60$

e) $f(43.5) = 43.5 + 40$
$= 83.5$

f) $f(100) = 100 + 40$
$= 140$

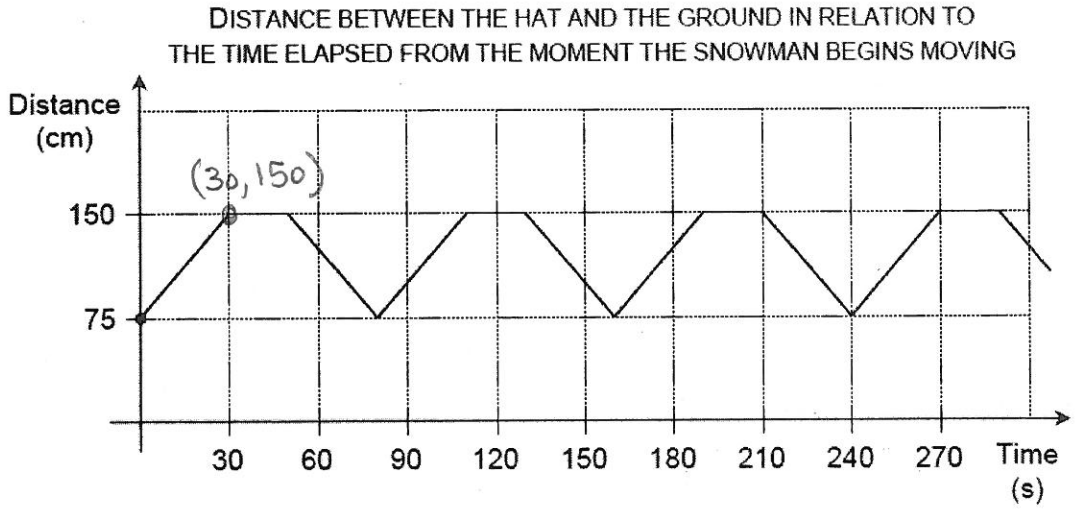
g) $f(0) = -2(0) + 50$
$= 50$

2.

Every winter, Nadine sets up an animated decoration on her front lawn. She uses a mechanical system to make her decorations move.

This year's decoration features a snowman holding a hat in his right hand. He starts by holding the hat at stomach level, raises it up onto his head and leaves it there for 20 seconds, and then takes it off and lowers it back down to stomach level. The snowman keeps repeating this set of movements.

The graph of periodic function f below represents the distance between the hat and the ground in relation to the time elapsed from the moment the snowman begins moving.



Nadine set the snowman in motion at 4:00 p.m. At that point, the snowman was holding his hat at stomach level. Exactly 7 minutes later, the snowman stopped moving because the mechanical system broke down.

What was the distance between the hat and the ground when the snowman stopped moving?

① $7\text{ min} \cdot \frac{60\text{ sec}}{\text{min}} = 420$ seconds

② Period $= \frac{240\text{ sec}}{3\text{ cycles}}$
$= 80$ seconds/cycles

Height at 420 seconds is the same as at $420 - 5$ cycles
$420 - 5(80)$
$= 20$ seconds

To find y when $x = 20$ seconds
find $y = ax + b$ for line through $(0,75)$ and $(30,150)$

$a = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow a = \frac{150-75}{30-0}$
$= \frac{75}{30}$
$= 2.5$

$b = 75$ (initial value)

let $x = 20$

$y = 2.5x + 75$
$y = 2.5(20) + 75$
$= 125$ cm

3.

Tony and Larry bought computer games at an electronics store. Each game costs a minimum of $20, excluding taxes.

This week's discount is posted at the entrance to the store.

THIS WEEK'S DISCOUNT
Get $3 off with every $20 spent, before taxes.

Tony bought a single game for $22.50, excluding taxes, and got a discount of $3.

Larry bought a single game for $45, excluding taxes, and got a discount of $6.

Fred, a friend who was with them, then came to the following conclusion:

If customers double the value of their purchase, the value of their discount will also double.

Is Fred right or wrong?
Explain why.

Table

x $ spent	y Discount
[0, 20[	0
[20, 40[	3
[40, 60[	6
[60, 80[	9
[80, 100[	12

If you spend 39$ Discount is 3$

Double 39$ is 78$ → Discount is 9$

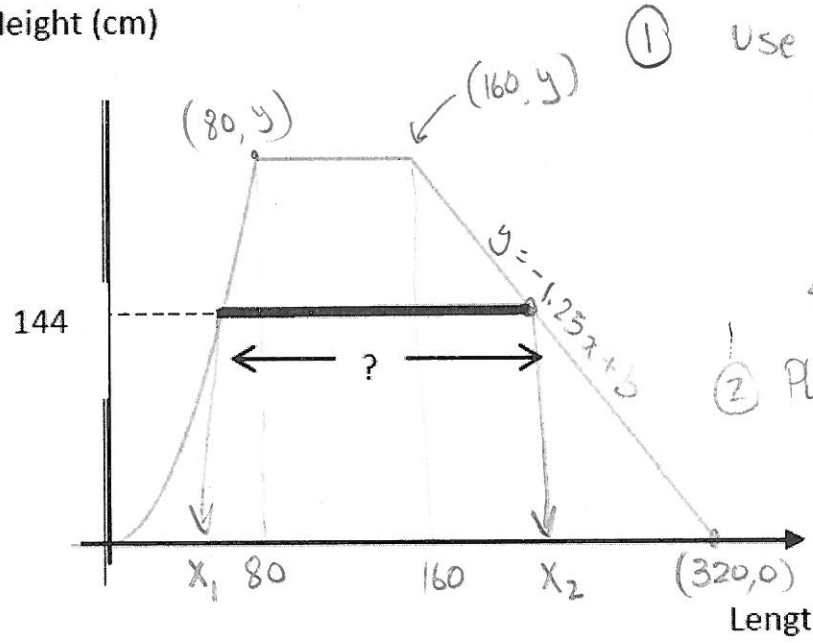
Fred is WRONG since 9$ is NOT double 3$!

4.

The graph below represents the outline of a skateboard ramp which corresponds to a piecewise function defined by:

$$f(x) = \begin{cases} ax^2 & \text{if } 0 \le x \le 80 \\ 256 & \text{if } 80 \le x \le 160 \\ -1.25x + b & \text{if } 160 \le x \le 320 \end{cases}$$

Put these on x-axis



① Use (320, 0) to solve for "b"

$y = -1.25x + b$

$0 = -1.25(320) + b$

$0 = -400 + b$

$400 = b$

② Plug in $x = 160$ to get y

$y = -1.25(160) + 400$

$y = 200$

For security purposes, a strip of reflective tape will be placed on the ramp at a height of 144 cm. ← y-value

③ Since (160, 200) then (80, 200) is on $y = ax^2$

use (80, 200) to solve for "a"

$200 = a(80)^2$

$200 = a(6400)$

$\frac{200}{6400} = a$

$0.03125 = a$

$y = 0.03125x^2$

What is the length of this piece of reflective tape?

④ Use $y = 144$ → get x_1

$y = 0.03125x^2$

$\frac{144}{0.03125} = \frac{0.03125x^2}{0.03125}$

$4608 = x^2$

$\pm 67.9 = x$ cm

$x_1 = 67.9$ cm

⑤ use $y = 144$ to get x_2

$y = -1.25x + 400$

$144 = -1.25x + 400$

$\frac{-256}{-1.25} = \frac{-1.25x}{-1.25}$

$204.8 \text{ cm} = x_2$

Length $= 204.8 - 67.9$

$= 136.9$ cm