QUESTION 1 1.1-MC-A

Answer and Solution:

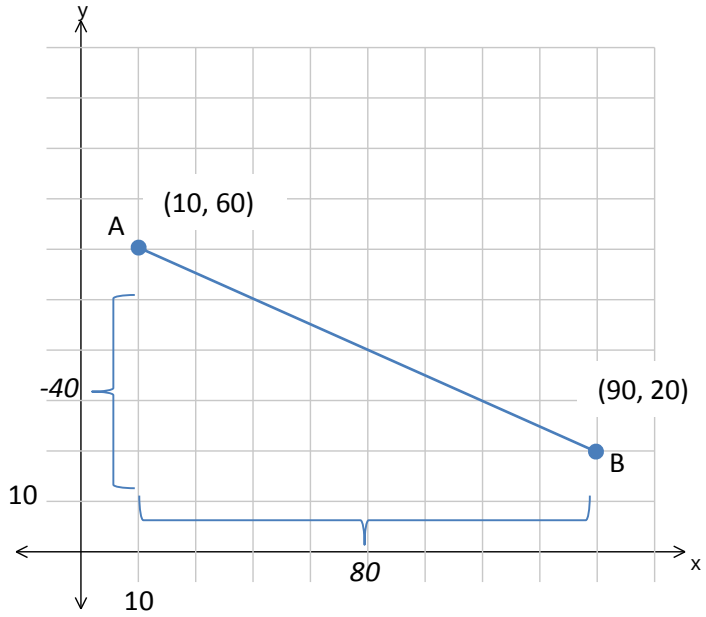
Find the slope between the two points either with the formula or visually from the graph:

- Formula from a table of values (or just the points):

x	*y*
10	60
90	20

$$\frac{20-60}{90-10}=\frac{-40}{80}=-\frac{1}{2}$$

- Rise over run from the diagram: $\frac{rise}{run}=\frac{-40}{80}=-\frac{1}{2}$



A) Incorrect: would be "run over rise"

B) **Correct**

C) Incorrect: Segment AB has a negative slope but $\frac{1}{2}$ indicates a positive slope

D) Incorrect: This is the positive version of "run over rise" so incorrect on two counts.

The answer is B.

Suggested Strategies:

I) Recognize the table of values is a series of coordinates… you need two sets to find the rule.

II) Choose any coordinate pair and label them x_1, y_1 and x_2, y_2

III) Find the slope between these coordinate pairs using the formula: $slope = \frac{y_2-y_1}{x_2-x_1}$

IV) Plug the slope into the formula $y = ax + b$

V) Substitute any of the (x, y) coordinate pairs from the table into the equation and solve for the initial value (b)

VI) If your slope or initial value is in fraction form, multiply each term by the LCM (least common multiple) of the two denominators

VII) Keeping in mind the signs, move all of the terms to one side of the equal sign.

Additional Resources:

Visions Volume 1, Section 1.1, p. 15 (Slope of a Segment)
Khan Academy video: http://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-relationships-functions/cc-8th-slope/v/slope-of-a-line
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Slope

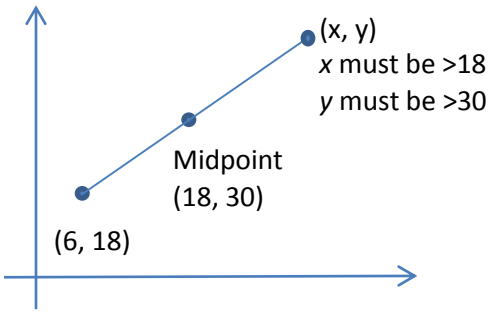
Answer and Solution:

Midpoint formula: (18, 30):$\left(\frac{(6+x_2)}{2}, \frac{(18+y_2)}{2}\right)$

$$18 = \frac{6+x}{2} \qquad 30 = \frac{18+y}{2}$$
$$36 = 6+x \qquad 60 = 18+y$$
$$30 = x \qquad 42 = y$$

The answer is (30, 42)

OR



The only answer that meets both conditions is D (30, 42).

A) (–6, 6) This is the answer you get if you mix up the endpoint and the midpoint.

B) (12, 24) This is the answer you get if you just plug the points into the midpoint formula.

C) (24, 12) This is if you just plug in the points into the midpoint formula and mix up the x and y.

D) (30, 42) This is the correct answer.

Suggested Strategies:

Use either the mid-point formula, remembering that you will have some algebra to do, or sketch it and see which answer makes sense.

Additional Resources:

Visions Volume 1, Section 1.1, p. 16 (Point of Division)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula

Answer and Solution:

Division point formula:

$$(x_p, y_p) = \left(x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1)\right)$$

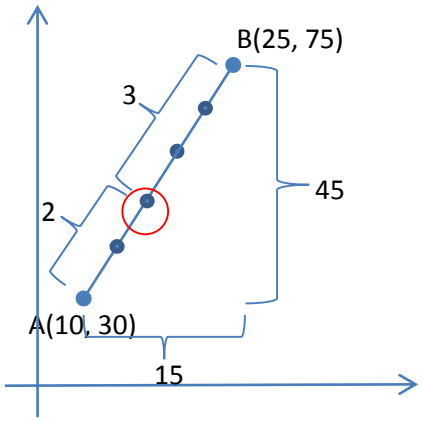
With $a = 3, b = 5, x_1 = 25, y_1 = 75, x_2 = 10, y_2 = 30$
OR $a = 2, b = 5, x_1 = 10, y_1 = 30, x_2 = 25, y_2 = 75$

Example for the x-coordinate with the first choice:

$$x_p = 25 + \frac{3}{5}(10 - 25)$$
$$x_p = 25 + \frac{3}{5}(-15)$$
$$x_p = 25 - 9 = 16$$

Repeat for the *y*-coordinate.

Here is the sketch:



A) Incorrect: this is the answer you get if you use $\frac{3}{8}$ instead of $\frac{3}{5}$.

B) Correct

C) Incorrect: this is the answer you get if you calculate the ratio from the wrong end (A to B instead of B to A).

D) Incorrect: this is the answer you get if you use $\frac{3}{8}$ instead of $\frac{3}{5}$ and went from the wrong end.

The answer is B.

Suggested Strategies:

- Be careful because the problem is stating the distance from B to A, not A to B. You can see that the distractors assume you might make this mistake.
- Determine whether the ratio given is part to part or part to whole. In this case it is part to whole. You can see that the distractors assume you might make the mistake of interpreting it as a part to part ratio.

You can solve the problem by using the distance formula – paying close attention to where you plug in your points. Remember the B to A. You could also use the other part of the ratio and use $\frac{2}{5}$ of the way from A to B. Don't let yourself be confused.

You can also sketch the points and see which answer(s) make sense.

Additional Resources:

Visions Volume 1, Section 1.1, p. 16 (Point of Division)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula

Answer and Solution:

Division point formula:

$$(x_p, y_p) = \left(x_1 + \frac{part}{whole}(x_2 - x_1), y_1 + \frac{part}{whole}(y_2 - y_1)\right)$$

With $\frac{part}{whole} = \frac{4}{4+1} = \frac{4}{5}$
And $x_1 = 200, y_1 = 800, x_2 = 1200, y_2 = 1600$

$$x_p = 200 + \frac{4}{5}(1200 - 200)$$
$$x_p = 200 + \frac{4}{5}(1000)$$
$$x_p = 200 + 800$$
$$x_p = 1000$$

$$y_p = 800 + \frac{4}{5}(1600 - 800)$$
$$y_p = 800 + \frac{4}{5}(800)$$
$$y_p = 800 + 640$$
$$y_p = 1440$$

Jim's house is situated at (1000, 1440).

Suggested Strategies:

Notice the keywords to see what kind of problem it is:

- Line, point
- Divides
- Ratio
- Coordinates

This is a division point question.

Determine whether the ratio given is part to part or part to whole. In this case it is part to part – which means you'll add the numbers to create the fraction in the formula.

Pay attention to the end from which the ratio is being measured – in this case from point A.

It always helps to make a sketch of the situation.

Additional Resources:

Visions Volume 1, Section 1.1, p. 16 (Point of Division)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/midpoint_formula

Answer and Solution:

$$d(A,C) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

A (-30,40) & C (20, 58)

$$d(A,C) = \sqrt{\big(20 - (-30)\big)^2 + (58 - 40)^2}$$
$$d(A,C) = \sqrt{(50)^2 + (18)^2}$$
$$d(A,C) = \sqrt{2500 + 324}$$
$$d(A,C) = \sqrt{2824}$$
$$d(A,C) \approx 53.1413 \text{ m}$$

B (90,35) & C (20, 58)

$$d(B,C) = \sqrt{(20 - 90)^2 + (58 - 35)^2}$$
$$d(B,C) = \sqrt{(-70)^2 + (23)^2}$$
$$d(B,C) = \sqrt{4900 + 529}$$
$$d(B,C) = \sqrt{5429}$$
$$d(B,C) \approx 73.6817 \text{ m}$$

$73.6817 - 53.1413 = 20.5404$ m

$\overline{BC}$ is <u>20.54</u> m longer than $\overline{AC}$.

Suggested Strategies:

The key word here is "longer" which implies length. And with the Cartesian plane (coordinates) as part of the question we'll want to use the distance formula.

Determine the distances we need: AC and BC (we don't need AB).

And then subtract to find the difference between the two distances calculated.

Additional Resources:

Visions Volume 1, Section 1.1, p. 15 (Distance between Two Points)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/midpoint-formula

Answer and Solution:

Distance between Bill's house and the water tower:

Bill's house: (–400, 200) Water tower: (0, 1100)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(0 - -400)^2 + (1100 - 200)^2}$$
$$d = \sqrt{(400)^2 + (900)^2}$$
$$d = \sqrt{160000 + 810000}$$
$$d = \sqrt{970000}$$
$$d \approx \mathbf{984.88\ m}$$

Coordinates of Alan's house:
Bill's house: (–400, 200)
School: (200, 400), the midpoint

$\frac{x_1+x_2}{2} = x_m$ $\frac{y_1+y_2}{2} = y_m$

$\frac{-400+x_2}{2} = 200$ $\frac{200+y_2}{2} = 400$

$-400 + x_2 = 400$ $200 + y_2 = 800$
$x_2 = 800$ $y_2 = 600$

Alan's house: (800, 600)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(0 - 800)^2 + (1100 - 600)^2}$$
$$d = \sqrt{(-800)^2 + (500)^2}$$
$$d = \sqrt{640000 + 250000}$$
$$d = \sqrt{890000}$$
$$d \approx \mathbf{943.40\ m}$$

Alan is correct; their houses are not the same distance from the water tower.

Suggested Strategies:

Begin by transferring information from the text onto the diagram.

To answer this question you have to calculate the distance between each of the houses and the water tower. For that, you need the three sets of coordinates; Bill's house, Alan's house and the water tower.

You are given the coordinates of the water tower (0, 1100) and Bill's house (–400, 200).

Using the coordinates of Bill's house and the school, you can determine the coordinates of Alan's house.

Additional Resources:

Visions Volume 1, Section 1.1, pp. 15-16
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/midpoint-formula

QUESTION 7 1.2-MC-A

Answer and Solution:

Formula for a linear equation: $y = ax + b$

$$slope\ (a) = \frac{3}{7}$$

Substitute coordinate values (7, 8) for x and y and solve for y-intercept b

$$y = \frac{3}{7}x + b$$
$$8 = \frac{3}{7}(7) + b$$
$$8 = 3 + b$$
$$8 - 3 = b$$
$$5 = b$$

$$y = \frac{3}{7}x + 5$$

The answer is A.

Suggested Strategies:

I) Plug the slope into the formula $y = ax + b$

II) Substitute the (x, y) coordinate pair into the equation and solve for the initial value or y-intercept (b)

III) Keeping in mind the signs, move all of the terms to one side of the equal sign.

IV) Re-write the formula with the slope and initial value to yield the equation of the line

Additional Resources:

Visions Volume 1, Section 1.2, p. 26 (Equation of Line from Slope and Intercepts)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/algebra--slope-and-y-intercept-intuition
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Point-Slope Form of a Line
- Slope-Intercept Form of a Line

QUESTION 8 1.2-MC-B

Answer and Solution:

Find the slope of *4x + 3y + 12 = 0*

$3y = -4x - 12$
$\frac{3y}{3} = \frac{-4x - 12}{3}$
$y = \frac{-4}{3}x - 4 \quad the\ slope\ is\ \frac{-4}{3}$

Find the perpendicular slope: $\frac{a}{b} \rightarrow \frac{-b}{a}$
The perpendicular slope is $\frac{3}{4}$ (options B and D are 'out')

Option A
$y = -\frac{3}{4}x + 2$

The slope is not $\frac{3}{4}$, option A is wrong.

Option C
$y = \frac{3}{4}x - 2$

The slope is equal to $\frac{3}{4}$, **option C is correct.**

The answer is C.

Suggested Strategies:

I) The word *perpendicular* in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes: $\frac{a}{b} \rightarrow \frac{-b}{a}$

II) Start by converting the equation from 'general' form to 'slope-intercept' form in order to get a better look at the slope.

III) Find the negative reciprocal of the slope from the equation given in the problem. This is the slope we are looking for in our multiple-choice answers.

IV) Remember that we only care about finding a perpendicular line in this problem, so we only need to worry about the slopes. Ignore the initial values altogether… they are only distractors here.

Additional Resources:

Visions Volume 1, Section 1.2, p. 27 (Perpendicular Line)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/e/line_relationships

QUESTION 9 1.2-MC-C

Answer and Solution:

Set the '*y*' value to 0 and solve for *x*.

$2x + 0 + 6 = 0$

$2x + 6 = 0$

$2x = -6$

$x = \frac{-6}{2}$

$x = -3$

The answer is A.

Suggested Strategies:

I) Remember that the *x*-intercept is the point on a graph where the line crosses the *x*-axis (*y* = 0)

II) Set *y* = 0 and solve for *x*.

Additional Resources:

Visions Volume 1, Section 1.2, p. 26 (Equation of a Line)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/algebra--equation-of-a-line

QUESTION 10 1.2-MC-D

Answer and Solution:

Convert the equation from general form to *y*-intercept form

$3x - 4y - 24 = 0$
$-4y = -3x + 24$
$\frac{-4y}{-4} = \frac{-3x + 24}{-4}$
$y = \frac{3x}{4} - 6\,, the\ slope\ of\ the\ parallel\ line\ must\ be\ \frac{3}{4}$

$y = ax + b$
$y = \frac{3}{4}x + b$
$7 = 0.75(-8) + b$
$7 = -6 + b$
$13 = b$

$y = \frac{3}{4}x + 13$

The answer is B.

Suggested Strategies:

I) Remember that parallel lines always have the same slope.

II) Convert the rule in the question from 'general' form to 'slope-intercept' form in order to find the slope. This slope will be the same in your new parallel line.

III) Plug the parallel slope (a) into the formula $y = ax + b$

IV) Substitute the coordinates of point P (-8, 7) into the new equation and solve for the initial value (*b*).

V)

Additional Resources:

Visions Volume 1, Section 1.2, p. 27 (Parallel Line)
Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/parallel-line-equation

Answer and Solution:

Convert the equation line L_2 from general form to standard form

$-4x + 5y - 10 = 0$

$5y = 4x + 10$

$\frac{5y}{5} = \frac{4x + 10}{5}$

$y = \frac{4x}{5} + 2\,, the\ slope\ of\ the\ line\ L_2\ is\ \frac{4}{5}$

Find the perpendicular slope of line L_1: $\frac{a}{b} \rightarrow \frac{-b}{a}$, $\frac{4}{5} \rightarrow \frac{-5}{4}\,, the\ perpendicular\ slope\ is \frac{-5}{4}$

$y = ax + b$

$y = \frac{-5}{4}x + b$

$15 = -1.2(12) + b$

$15 = -14.4 + b$

$30 = b$

$y = \frac{-5}{4}x + 30$

Set the '*y*' value to 0 and solve for *x*.

$0 = \frac{-5}{4}x + 30$

$-30 = \frac{-5}{4}x$

$-30\left(\frac{4}{-5}\right) = x$

$\boldsymbol{24 = x}$

The *x*-intercept of the line L_1 is 24

Suggested Strategies:

I) Remember that an *x*-intercept is the point at which a line crosses the *x*-axis. The *y*-value of this coordinate must be equal to 0 (*y* = 0) at this point.

II) Start by converting the equation from general form to standard form in order to get a better look at the slope.

III) The word *perpendicular* in this problem should immediately cause you to write the negative reciprocal rule for perpendicular slopes: $\frac{a}{b} \rightarrow \frac{-b}{a}$.

IV) Since we are looking for the line that is *perpendicular* to −4x + 5y – 10, we'll need the negative reciprocal ($\frac{-b}{a}$) of the slope from the equation given in the problem.

V) Use the perpendicular slope in a new '*y* = *ax* + *b*' rule.

VI) Substitute the coordinates of point P (12, 15) into the new equation and solve for the initial value (*b*).

VII) Once you've got your rule for the perpendicular line finished, find the *x*-intercept by making *y* = 0 and solving for *x*.

Additional Resources:

Visions Volume 1, Section 1.2, pp. 26-27

Khan Academy video: http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/more-analytic-geometry/v/parallel-line-equation

QUESTION 12 1.2-SA-B

Answer and Solution:

Line L_1 :

$$y = \frac{4x}{3} - 3$$

Line L_2 :

$$a = \frac{-3}{4}$$

Substitute point (2, 5) in for the *x* and *y* to solve for "*b*".

$$y = \frac{-3x}{4} + b$$

$$y = \frac{-3(2)}{4} + b$$

$$5 = \frac{-6}{4} + b$$

$$b = 5 + \frac{3}{2} = 6.5$$

$$y = \frac{-3x}{4} + 6.5$$

The equation of line L_2 is $y = \frac{-3x}{4} + 6.5$

Suggested Strategies:

- Use the negative reciprocal of the slope of line L_1 to find the slope of line L_2.

- Use the function form of the equation, $y = ax + b$, with point (2, 5) and the new slope.

Additional Resources:

Visions Volume 1, p. 27 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Systems of Linear Equations

Answer and Solution:

Algebraically:

A) *x*-intercept

$y = 0$

$8x + 6y + 12 = 0$

$8x + 6(0) + 12 = 0$

$8x = -12$

$x = \frac{-12}{8} = \frac{-3}{2}$

$x = \frac{-3}{2}$

B) *y*-intercept

$x = 0$

$8(0) + 6y + 12 = 0$

$6y + 12 = 0$

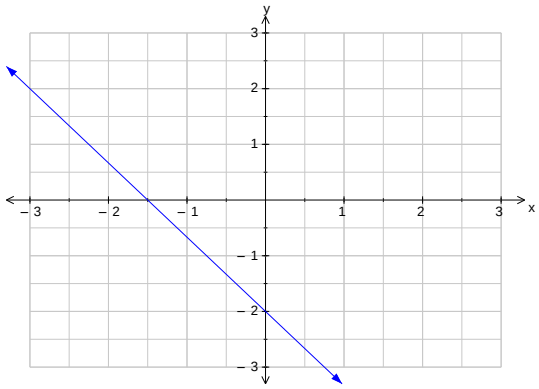
$6y = -12$

$y = \frac{-12}{6} = -2$

$y = -2$

Graphically:

$$8x + 6y + 12 = 0 \rightarrow y = -\frac{4}{3}x - 2$$



The x-intercept is $-\frac{3}{2}$.

The y-intercept is –2.

Suggested Strategies:

When finding the intercepts, the other coordinate is 0:

x-intercept means $y = 0$
y-intercept means $x = 0$.

You can also solve this question graphically by changing the equation into function form and plotting the y-intercept and slope.

Additional Resources:

Visions Volume 1, p. 26 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

Answer and Solution:

Find the coordinates of the point where the car breaks down

Division point 2/3 of the way between A and B

Division point :$\left(x_1 + \frac{a}{b}(x_2 - x_1), y_1 + \frac{a}{b}(y_2 - y_1)\right)$

A (–24, –39)
B (30 , 33)

$$\left(-24 + \frac{2}{3}\big(30 - (-24)\big), -39 + \frac{2}{3}(33 - (-39))\right)$$

$$\left(-24 + \frac{2}{3}(54), -39 + \frac{2}{3}(72)\right)$$

Call the position of the car point C

$$\mathbf{C}\ (\mathbf{12}, \mathbf{9})$$

Find the slope of the rule for the line the car travels

Formula for a linear equation: $\boldsymbol{y = ax + b}$

Coordinates used to find the rule:

A (–24,–39)
B (30 , 33)

$$slope\ (a) = \frac{y_2 - y_1}{x_2 - x_1}$$

$$slope\ (a) = \frac{33 - (-39)}{30 - (-24)} = \frac{72}{54} = \mathbf{\frac{4}{3}}$$

Find the equation for the tow-truck's path

The tow-truck's path will be perpendicular (negative reciprocal slope) and passes through point C (12, 9) , *the car*

$\frac{a}{b} \to \frac{-b}{a}$, $\frac{4}{3} \to \frac{-3}{4}$, the tow-truck's slope is $\frac{-3}{4}$

$\boldsymbol{y = ax + b}$, passing through (12, 9)

$$y = -\frac{3}{4}x + b$$

$$9 = \frac{-3}{4}(12) + b$$

$$9 = -9 + b$$

$$18 = b$$

Suggested Strategies:

I) Recognize that the question is asking for a distance between two points, the car and the garage. This problem requires us to first find and then use those coordinates.

II) Start by using the division point formula to find the coordinates of the car when it breaks down.

III) Then find the coordinates of the garage,
   a. we know that it is on the *x*-axis (so the *y*-coordinate is = 0)
   b. We know that it is on the path that is perpendicular to AB

IV) Find the slope of AB so we can use its negative reciprocal to define the slope of the line between the car and the garage.

V) Plug the (*x*, *y*) coordinates of the car into the formula for the tow-truck's path, then solve for the initial value to complete the equation for the tow-truck's path.

$$\boldsymbol{y = \frac{-3}{4}x + 18}$$

VI) Using the equation of the line for the tow-truck's path, set the *y*-coordinate to 0 (since the garage is on the *x*-axis) and solve for *x*.

VII) Use the distance formula along with the coordinates of the car and garage to find your final answer.

<u>Find the coordinates of the Garage on the x-axis</u>

Set the '*y*' value to 0 and solve for *x*.

$$\boldsymbol{y = \frac{-3}{4}x + 18}$$
$$\boldsymbol{0 = \frac{-3}{4}x + 18}$$
$$\boldsymbol{-18 = \frac{-3}{4}}$$
$$\boldsymbol{-18\left(\frac{4}{-3}\right) = x}$$

24 = x

The coordinates of the garage are: (24, 0)

<u>Find the distance from the garage to the car.</u>

Garage (24, 0)
Car (12, 9)

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$distance = \sqrt{(12 - 24)^2 + (9 - 0)^2}$$
$$distance = \sqrt{(-12)^2 + (9)^2}$$
$$distance = \sqrt{144 + 81}$$
$$distance = 15$$

Final answer: the distance the tow-truck must travel from the garage to the car is 15 km.

Additional Resources:

Visions Volume 1, Section 1.2, pp. 26-27
Visions Volume 1, Section 1.1, pp. 15-16

QUESTION 15 1.3-MC-A

Answer and Solution:	**Suggested Strategies:**

The slopes (a) are the same and the y-intercepts (b) are different; if graphed you would see that the lines are parallel and never intersect.
 	
The answer is C.
	Solve the system.
	
	Check to see if the slopes are the same;
	- If not, there will be one solution;
	- If the slopes are the same, check to see if the y-intercepts are the same
	  ○ If they are the same, there is an infinite number of solutions since they are the same line;
	  ○ If they are not, they are parallel lines and the system has no solution.

Additional Resources:

Visions Volume 1, p. 40 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Systems of Linear Equations

Answer and Solution:

$2x - 5y + 12 = 0$
$x - 3y = 4$

Substitution method (since it is easy to isolate x.)

$x - 3y = 4$
$x = 3y + 4$

$2x - 5y + 12 = 0$
$2(3y + 4) - 5y + 12 = 0$
$6y + 8 - 5y + 12 = 0$
$y + 20 = 0$
$y = -20$

$x = 4 + 3y$
$x = 4 + 3(-20)$
$x = 4 - 60$
$x = -56$

$(-56, -20)$

Check:
$2x - 5y + 12 = 0$
$2(-56) - 5(-20) + 12 = 0$
$-112 + 100 + 12 = 0$ True

$x - 3y = 4$
$(-56) - 3(-20) = 4$
$-56 + 60 = 4$ True

Both are true, so $(-56, -20)$ is the correct solution.

The answer is C.

Suggested Strategies:

Method 1:

Solve the system of equations by the method of your choice – this one lends itself to substitution.

Method 2:

Check by substituting each possible answer into the two equations to verify which point is a possible solution.

$(-41, -14)$
$2x - 5y + 12 = 0$
$2(-41) - 5(-14) + 12 = 0$
$-82 + 70 + 12 = 0$
$0 = 0$
$x - 3y = 4$
$-41 - 3(-14) = 4$
$-41 + 52 = 4$
$11 \neq 4$

$(-44, -20)$
$2x - 5y + 12 = 0$
$2(-44) - 5(-20) + 12 = 0$
$-88 + 100 + 12 = 0$
$24 \neq 0$

$(-56, -12)$
$2x - 5y + 12 = 0$
$2(-56) - 5(-12) + 12 = 0$
$-112 + 60 + 12 = 0$
$-40 \neq 0$

Additional Resources:

Visions Volume 1, p. 39 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Systems of Linear Equations

QUESTION 17 1.3-MC-C

Answer and Solution:

x = cost of a chocolate chip cookie
y = cost of a peanut butter cookie

$3x + 4y = 5.65$
$5x + 7y = 9.70$

$-5(3x + 4y = 5.65)$
$3(5x + 7y = 9.70)$

$-15x - 20y = -28.25$
$15x + 21y = 29.10$
$y = 0.85$

cost of a peanut butter cookie = \$0.85

A) 70 cents

B) 75 cents – is cost of chocolate chip cookies

C) 80 cents

D) **85 cents**

The answer is D.

Suggested Strategies:

Set up a system of equations and solve it.

- Define your variables
- Write your equations
- Choose a method (this one suggests elimination method but the other methods work as well.)
- Interpret your answer correctly by seeing which variable represents the cost of the peanut butter cookie.

Additional Resources:

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Systems of Linear Equations

Answer and Solution:

x = cost for long cabinet
y = cost for short cabinet

Client A: $7x + 4y + 120 = 1840$ OR $7x + 4y = 1720$
Client B: $9x + 8y + 190 = 2630$ OR $9x + 8y = 2440$
Client C: $11x + 2y + 170 = ?$

By elimination method:

Step 1)

$-2(7x + 4y = 1720)$
$9x + 8y = 2440$

$-14x - 8y = -3440$
$9x + 8y = 2440$
$-5x = -1000$

$x = \frac{-1000}{-5}$
$x = 200$

Step 2)

$7x + 4y = 1720$
$7(200) + 4y = 1720$
$1400 + 4y = 1720$
$4y = 1720 - 1400$
$4y = 320$
$y = \frac{320}{4} = 80$

$x = 200$
$y = 80$

Client C: $11x + 2y + 170 = ?$
$11(200) + 2(80) + 170 =$
$2200 + 160 + 170 = 2530$
$2530

Client C is correct. His total cost will be lower than client B's, since it is $2530 compared to $2630.

Suggested Strategies:

This is a "system of equations" question.

In order to find the cost for Client C, you need to know how much each type of cabinet costs.

Use the information given for the other two clients to find those costs.

- Define your variables,
- Set up two equations in two unknowns,
- Solve the system,
- Use the solution to find the cost for Client C

Note: if you don't show any work and just check one of the boxes, you will get zero.

Additional Resources:

Visions Volume 1, pp. 39-40 (Mathematical Knowledge Summary)
Khan Academy video: http://www.khanacademy.org/math/algebra/systems-of-eq-and-ineq/fast-systems-of-equations/v/solving-systems-of-equations-by-elimination
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Systems of Linear Equations

QUESTION 25 2.1-MC-A

Answer and Solution:

1. Mean= (21+21+21+23+23+23)÷6
   Mean= 22

2. |21-22| = |-1| = 1
   |21-22| = |-1| = 1
   |21-22| = |-1| = 1
   |23-22|= 1
   |23-22|= 1
   |23-22|= 1

3. Mean deviation= (1+1+1+1+1+1)÷ 6
   Mean deviation= 1

A) 0 – if you didn't take the absolute value of the differences

B) **1 – correct**

C) 2 – unlikely, but just in case you take the difference between the two repeated values

D) 6 – if you forget to divide by 6

The answer is B.

Specific Strategies:

1. Calculate the mean of the set of data.
2. Subtract the mean from each value in the set of data and determine its absolute value. (Remember absolute values can't be negative.)
3. Calculate the mean of the deviations.

Additional Resources:

Visions Volume 1, p. 81
http://www.wikihow.com/Calculate-Mean-Deviation-About-Mean-(for-Ungrouped-Data)

Answer and Solution:

I. The mean, median, and range are measures of central tendency. False – range is a measure of dispersion.

II. Percentile rank is a measure of dispersion. False – percentile is a measure of position.

III. **The mean deviation and range are measures of dispersion. True.**

IV. The mean deviation is a measure of position. False – mean deviation is a measure of dispersion.

The answer is B.

Suggested Strategies:

It is important to remember your vocabulary. Instead of blindly calculating mean, range etc. Try to think about why you are doing them and what the result represents. This goes for any stats question.

Recall:

- The measures of central tendency are mean, median, and mode.
- The measures of position are percentile rank
- Range and mean deviation are measures of dispersion.

Additional Resources:

Visions Volume 2, pp. 81-82
Khan Academy video: https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-probability-statistics/cc-7th-central-tendency/v/statistics-intro--mean--median-and-mode

QUESTION 27 2.1-MC-C

Answer and Solution:

1. There are 42 values in the stem and leaf plot
2. Apply the formula: $\frac{percentile}{100} \times number\ of\ values$

$$\frac{70}{100} \times 42$$

$$0.70 \times 42$$

$$29.4$$

3. Always round down the result (round to 29).
4. The answer represents the POSITION from the bottom (lowest result) of the data value you wish to find.

	Number of sit-ups
2	0 1 1 2 2 8 9
3	2 2 3 4 5 6 6 8 9
4	1 1 2 3 4 4 4 5 6 7 8
5	0 1 2 2 5 6 6 7 8 8 8
6	2 4 6 6

29^{th} value is 51

A) This is the result when you make both mistakes as described in B) and D).

B) This is the result when you count from the wrong end.

C) This is correct.

D) This is the result when you round up instead of down – it's the 30^{th} position.

The answer is C.

Specific Strategies:

Read the stem and leaf plot correctly.

Do the calculations completely before selecting your answer.

The percentile is given which means you need to work backwards to find the data value.

1. Find the total amount of values
2. Find the position from the bottom by applying the formula
3. Locate that position according to value found in step 3.

Additional Resources:

Khan Academy Video: https://www.khanacademy.org/math/arithmetic/applying-math-reasoning-topic/reading_data/e/reading_stem_and_leaf_plots
http://www.purplemath.com/modules/stemleaf2.htm

QUESTION 28 2.1-MC-D

Answer and Solution:

Answer using the formula:

$$\frac{number\ of\ data\ values\ below\ x + \frac{number\ of\ data\ values\ equal\ to\ x}{2}}{total\ number\ of\ data\ values} \times 100$$

$$\frac{146 + \frac{3}{2}}{305} \times 100 = \frac{146 + 1.5}{305} \times 100 = \frac{147.5}{305} \times 100$$

$$\approx 48.36\ round\ up\ to\ \mathbf{49}$$

A) 47 – if you don't take into account the 3 values at 50

B) 48 – if you round down or don't take into account the 3 values at 50

C) 49 – correct

D) 50 – if you just take the value itself

The answer is C.

Suggested Strategies:

The question is asking for percentile, so you need the formula which gives you the percentile of a data value.

Be careful:

Do not use the formula for finding a data value when the percentile is given!

Remember to round to the next whole number (always UP!)

Additional Resources:

Visions Volume 1, Section 2.1, pp. 76-88

Answer and Solution:

It is suggested to organize your work in a table.

1. The mean of the set of data is 19.55.

2. See the third column of the table.

Value	Mean	|Difference of value from mean|
41	19.55	21.45
17	19.55	2.55
25	19.55	5.45
9	19.55	10.55
20	19.55	0.45
12	19.55	7.55
11	19.55	8.55
21	19.55	1.45
20	19.55	0.45
Total:	58.45	

3. The mean deviation is:

$$\text{Mean deviation} = \frac{58.45}{9} = 6.49$$

The mean deviation for the set of data is <u>6.49</u>.

Specific Strategies:

1. Calculate the mean of the set of data
2. Subtract the mean from each value in the set of data
3. Calculate the mean of the deviations

Remember to take the absolute value of each deviation NO NEGATIVE VALUES!

Additional Resources:

Visions Volume 1, p. 81
http://www.mathsisfun.com/data/mean-deviation.html

Answer and Solution:

Solution A

Percentile rank=

$$\frac{total\ number\ of\ values\ below + \frac{total\ number\ of\ values\ equal\ to}{2}}{total\ number\ of\ values} \times 100$$

$$\frac{103+\frac{2}{2}}{137} \times 100 = 75.9$$

Round up to the next integer: 76th

Solution B

Percentile rank =

$$\frac{percentile\ rank}{100} \times \#\ of\ values = position$$

$$\frac{60}{100} \times 137 = 82.2\ or\ 82\ from\ the\ bottom$$

or

$$137 - 82 = 55\ runners\ finished\ ahead\ of\ him$$

He is the 56th runner from the top/82nd from the bottom with a time of 30:39.

Note: Your answer might be different if you use a different percentile formula.

A) The percentile rank of a runner finishing with 28:45 is <u>the 76th</u>.

B) The finishing time of a runner ranked in the 60th percentile is <u>30:39</u>.

Suggested Strategies:

Is the question asking for the percentile rank or the value at a specific position?

This will help determine which equation to use.

Remember a longer time is located at the bottom of the list.

1. First make sense of the table and notice that the numbers increase as you go down and across. There are 20 rows – use that fact as you count positions.
2. You'll notice there are two runners with a time of 28:45 and that they were faster than 103 of the other runners.
3. Apply the formula for finding percentile.
4. Round up to the next integer
5. Reverse the procedure to find the time for the 60th percentile.

Additional Resources:

Visions Volume 1, p. 81-82

Answer and Solution:

Criteria 1:

Swimmers that meet qualification 1:

$$\frac{60}{100}(20) = 12$$

This means 12 swimmers are at or below the 60th percentile.

If 12 of the 20 swimmers are at or below the 60th percentile, then 8 are above it.

Criteria 2:

Mean of the distribution: 19.77 (sum of all values ÷ 20)

Mean deviation of the distribution: 0.557 (sum of all mean deviations ÷ 20)

$$|18.56 - 19.77| = 1.21$$
$$|18.7 - 19.77| = 1.07$$
$$\vdots$$
$$|21.1 - 19.77| = 1.33$$

Sum of deviations = 11.14
11.14 ÷ 20 = 0.557

Swimmers that meet qualification 2.
PBT $\leq$ 20 – MD
PBT $\leq$ 20 – 0.557
PBT $\leq$ 19.443

They are: 18.56, 18.7, 18.9, 18.95, 19.2, 19.25, 19.26

Seven (7) swimmers will earn a spot on the National team.

Specific Strategies:

Remember that the lowest scores are the best scores. Therefore, the values in this distribution are given in order of best to worst, not from worst to best (which is what we usually see). This makes finding the percentile a little bit tricky.

Make sure that your answer takes into account the requirement to meet or exceed both of the evaluations that are used to select team members.

Additional Resources:

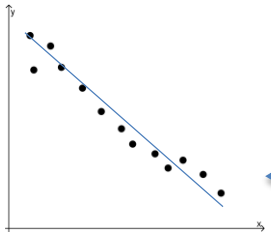
Visions Volume 1, pp. 81-82

QUESTION 32 2.2-MC-A

Answer and Solution:

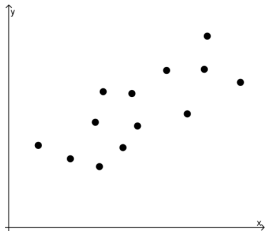
Answers B) and D) can be ruled out since the points are quite far apart.

A)

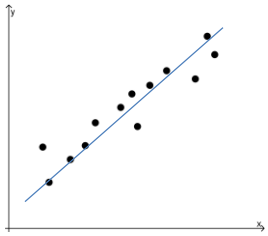


The points are closest to forming a straight line but strongest in A)

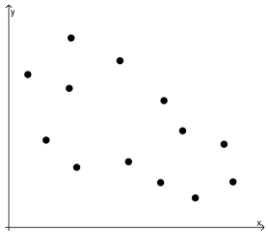
B)



C)



D)



The answer is A.

Specific Strategies:

The scatterplot that shows a distribution of points closest to forming a straight line has the strongest correlation.

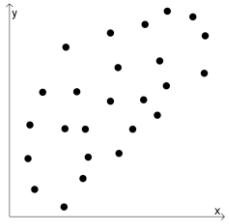
The strength of a correlation is not related to its direction (or sign).

Additional Resources:

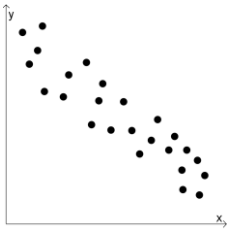
Visions Volume 1, Section 2.2, pp. 93-95
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

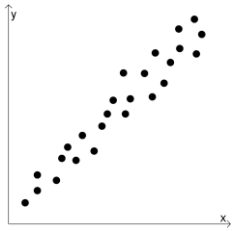
Answer and Solution:

A) 

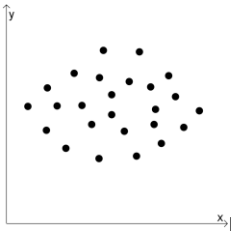
a slight positive line, so not 0

B) 

a fairly significant clustering around a negative line

C) 

a rather tight clustering around a positive line

D) 

no tendency towards a line

The scatterplot that shows a distribution of points with no clear direction and furthest from forming a straight line indicates a zero correlation.

The answer is D.

Suggested Strategies:

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-95
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

Answer and Solution:

DISTANCE (m) / AGE (years)	[2, 4[	[4, 6[	[6, 8[	[8, 10[	[10, 12[
[10, 11[	3	3	3	3	3
[11, 12[	3	3	3	3	3
[12, 13[	3	3	3	3	3
[13, 14[	3	3	3	3	3
[14, 15[	3	3	3	3	3

In a table, the closer the data is to the diagonal, the stronger the correlation.

All values are 3 so data is not clustered on the diagonal.

In this case, the data is evenly spread out, thus indicating no correlation.

A) The correlation is positive – false: data is not clustered around a diagonal from top left to bottom right.

B) The correlation is negative – false: data is not clustered around a diagonal from bottom left to top right.

C) The correlation is perfect – false: given an age, there is no way to predict the distance from the stage.

D) **The correlation is zero – true:** given an age, there is no way to predict the distance from the stage.

The answer is D.

Specific Strategies:

The closer the data is to the diagonal, the stronger the correlation.

A correlation may be zero, weak, moderate, strong or perfect!

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-94

QUESTION 35 2.2-SA-A

Answer and Solution:

x \ *y*	[0,1[	[1,2[	[2,3[	[3,4[	[4,5[
1	2	0	0	0	0
2	0	2	0	0	0
3	0	3	2	2	0
4	0	0	0	5	2
5	0	0	0	1	1

The answer is:

Strength	Direction
☐ Weak	☑ Positive
☑ Strong	☐ Negative

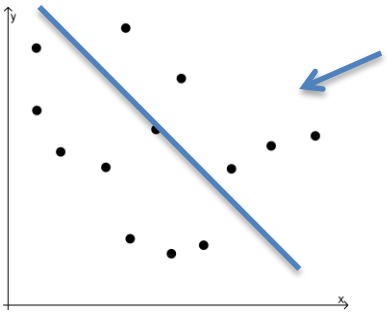
Suggested Strategies:

- In a table, the closer the data is to the diagonal, the stronger the correlation.
- If the diagonal slopes downward, then the correlation is positive – because as *x* increases, so does *y*.

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-94

Answer and Solution:



- The points appear to form a descending line (negative direction)
- The points are very scattered far apart (weak correlation)

The answer is:

Strength	Direction
☑ Weak	☐ Positive
☐ Strong	☑ Negative

Specific Strategies:

Look at how far or close the points are relative to each other

- The closer the points are to forming a straight line, the stronger the correlation is
- A positive slope means a positive correlation
- A negative slope means a negative correlation

Additional Resources:

Visions Volume 1, Section 2.2, pp. 93-95
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

QUESTION 37 2.3-MC-A

Answer and Solution:

A) –0.81 , 0.74 , 0.39 , –0.27
is from strongest to weakest

B) –0.27 , 0.39 , 0.74 , –0.81
Correct

C) –0.81 , –0.27 , 0.39 , 0.74
is from smallest number to greatest (keeps the sign)

D) 0.74 , 0.39 , –0.27 , –0.81
is from largest number to smallest number (keeps the sign)

The answer is B.

Specific Strategies:

Ignore the sign
(+ or –)!

- Order the coefficients (without the sign) in increasing order.
- The coefficient closest to 0 is the weakest.
- The coefficient closest to 1 is the strongest.

Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

QUESTION 38 2.3-MC-B

Answer and Solution:

A) −0.75 – don't mistake the smallest number with the weakest correlation

B) −0.45 – is further from 0 than 0.16

C) 0.16 – correct

D) 0.83 – this is the strongest correlation shown

The answer is C.

Specific Strategies:

Ignore the + or – signs!

The coefficient closest to 0 is the weakest

Additional Resources:

Visions Volume 1, Section, 2.3 p. 110
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

QUESTION 39 2.3-MC-C

Answer and Solution:

A) –1.00 – correct (perfect correlation can be positive or negative)

B) –0.10 – in case you misplace the decimal

C) 0.00 – in case you mix up "perfect" and "no correlation"

D) 0.99 – really close, but not perfect.

The answer is A.

Specific Strategies:

Ignore the + or – signs!

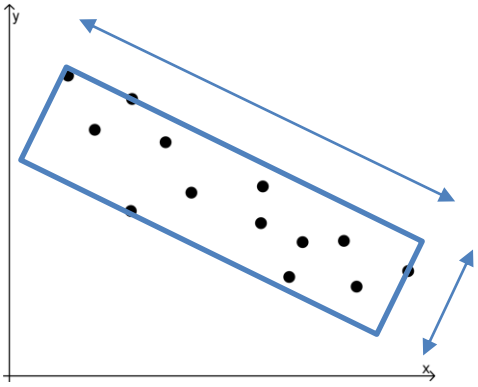
A perfect correlation is equal to 1 or –1

Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

Answer and Solution:



$$r = \pm\left(1 - \frac{short\ side}{long\ side}\right)$$

$$r = -\left(1 - \frac{1.4\ cm}{5.3\ cm}\right)$$

$$r = -(0.74)$$

Since the slope is negative, the correlation coefficient should also be negative

The linear correlation coefficient is –0.74.

(Because of slight differences in measurement, your answer could be anywhere between –0.82 and –0.66.)

Specific Strategies:

1. Encase the points within the tightest rectangle possible.
2. Measure the dimensions of the rectangle.
3. Estimate the linear correlation coefficient using the formula:

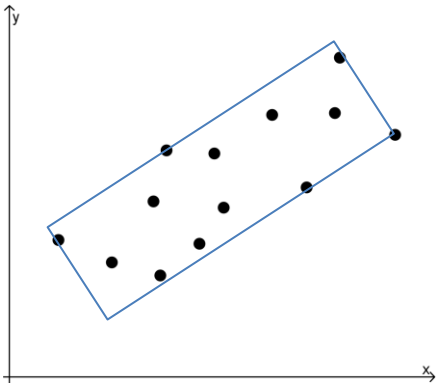
$$r = \pm\left(1 - \frac{short\ side}{long\ side}\right)$$

Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

Answer and Solution:



$$r = \pm\left(1 - \frac{short\ side}{long\ side}\right)$$

$$r = +\left(1 - \frac{1.5\ cm}{4.5\ cm}\right)$$

$$r = +(0.67)$$

Since the slope is positive, the correlation coefficient should also be positive.

The linear correlation coefficient is 0.67.

(Because of slight differences in measurement, your answer could be anywhere between 0.59 and 0.75)

Specific Strategies:

Encase the points with a rectangle. Measure the dimensions of the rectangle. Estimate the linear correlation coefficient using the formula:

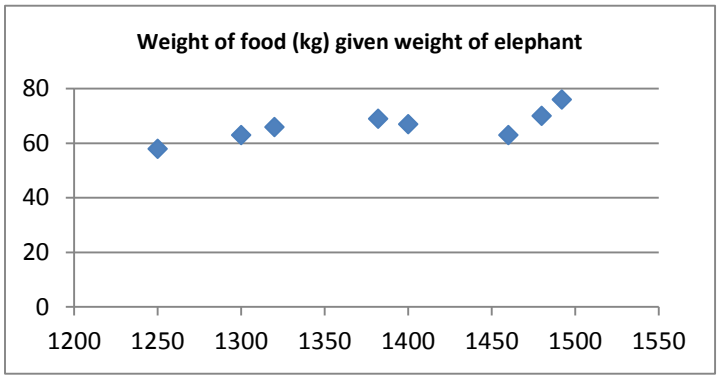
$$r = \pm\left(1 - \frac{short\ side}{long\ side}\right)$$

Additional Resources:

Visions Volume 1, Section 2.3, p. 110
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Correlation
- Trends in Scatter Plots
- Scatter Plots-Activity

Answer and Solution:



Perform a linear regression using the data in the table.

Mayer line method:

Let x be the weight of an elephant, and y be the weight of the food

Split data in half:

x	1250	1300	1320	1382		1400	1460	1480	1492
y	58	63	66	69		67	63	70	76

$$\text{average x first half} = \frac{1250+1300+1320+1382}{4} = 1313$$

$$\text{average x secondhalf} = \frac{1400+1460+1480+1492}{4} = 1458$$

$$\text{average y first half} = \frac{58+63+66+69}{4} = 64$$

$$\text{average y secondhalf} = \frac{67+63+70+76}{4} = 69$$

Point 1 (1313, 64)

Point 2 (1458, 69)

$$a = \frac{69-64}{1458-1313} = 0.0344$$

$$b = 64 - (0.0344)(1313) = 18.8328$$

Linear regression rule: $y = 0.0344x + 18.8328$

Find the value of y when $x = 1600$

$y = 0.0344(1600) + 18.8328$

$= 73.8728$

A 1600 kg elephant would be given 73.9 kg of food. Your answer might vary.

Suggested Strategies:

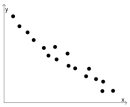
This is a case where we use data to come up with a rule in order to predict, or in this case extrapolate. Since the data appears to be linear choose a method for determining a regression line. You can use

- the Mayer line method (shown here) or
- the Median-median method, or use
- the regression line from your calculator or
- draw it by hand

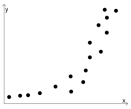
Additional Resources:

Visions Volume 1, Section 2.4, pp. 123 – 124.
http://www.purplemath.com/modules/scattreg2.htm

Answer and Solution:

A) 

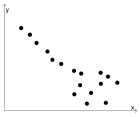
This data is clustered along a line

B) 

This data is clustered along a curve (not linear)

C) 

This data has a large gap and therefore a straight linear correlation can't be assumed

D) 

This data is clustered in one area rather than along a line

The answer is A.

Suggested Strategies:

The graph that shows data points that closely form a straight line yields the best interpretations for linear correlation

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines

QUESTION 44 2.4-MC-B

Answer and Solution:

B) – the data points in this graph are closest to forming a line.

The answer is B.

Suggested Strategies:

The graph showing the strongest correlation, whether positive or negative, would demonstrate the strongest statistical link between two variables. A strong statistical link leads to better prediction.

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines
http://www.purplemath.com/modules/scattreg2.htm

QUESTION 45 2.4-MC-C

Answer and Solution:

A) False: The correlation between the two variables is strong and **not** positive.

B) The correlation between the two variables is strong and negative.

C) False: The correlation between the two variables is **not** weak and positive.

D) False: The correlation between the two variables is **not** weak and negative.

The answer is B.

Suggested Strategies:

The options are all wordy but you will see that they are almost exactly the same; look for the differences – you might want to use a highlighter…

The correlation coefficient of -0.93 is close to -1, this results in a correlation described as negative and strong.

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines
http://www.dummies.com/how-to/content/how-to-interpret-a-correlation-coefficient-r.html

QUESTION 46 2.4-MC-D

Answer and Solution:

A) 0.32 – close to 0 and positive

B) 0.87 – positive but not close to 0

C) −0.26 – closest to 0 but negative

D) −0.91 – negative and not close to 0

The answer is A.

Suggested Strategies:

A weak and positive correlation is best represented by a correlation coefficient value that is positive and much closer to 0 than to 1.

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines

QUESTION 47 2.4-MC-E

Answer and Solution:

A) Strong and positive

B) Strong and negative

C) Weak and positive

D) Weak and negative

The answer is D.

Specific Strategies:

This scatterplot shows data points that trend downward suggesting a negative correlation.

This scatterplot also shows data points that are spread apart rather than close together (along a line) suggesting a weak correlation.

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines

QUESTION 48 2.4-MC-F

Answer and Solution:

A) 0.29 – positive by not high enough so suggest a strong correlation

B) 0.83 – positive and high enough to suggest a strong correlation

C) −0.45 – a negative correlation, and not very strong

D) −0.79 – a negative correlation, even though it is fairly strong.

The answer is B.

Suggested Strategies:

This scatterplot shows data points that trend upward therefore you are looking for a positive correlation coefficient.

This scatterplot also shows data points that are close together (along a line) rather than spread out so you are looking for a correlation coefficient that suggests a strong correlation.

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Khan Academy Video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see: Fitting a Line to Data, Estimating the Line of Best Fit
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving Using Trend Lines

Answer and Solution:

With the Mayer line method:

x represents the foot length in centimetres
y represents the height in centimetres

Since the foot lengths are already in ascending order this step has been done for us.

$x_1 = \frac{22+22+23+23.5+24+24+24.5+25+25+25.5}{10} = 23.85$

$x_2 = \frac{25.5+25.5+26+27+27.5+28+28+28.5+29+29.5}{10}$

$= 27.45$

$y_1 = \frac{154+151+155+165+160+158+165+161+163+164}{10}$

$= \frac{1596}{10} = 159.6$

$y_2 = \frac{170+173+167+174+175+176+183+185+190+186}{10}$

$= \frac{1779}{10} = 177.9$

$a = \frac{177.9 - 159.6}{27.45 - 23.85} = \frac{183}{3.6} = 5.08$

$y = 5.08x + b$ using either point (23.85, 159.6)
$159.6 = 5.08(23.85) + b$ or (27.45, 177.9)
$159.6 = 121.16 + b$
$b = 38.44$
$y = 5.08x + 38.44$

Marco: $181 = 5.08x + 38.44$
$181 - 38.44 = 5.08x$
$142.56 = 5.08x$
$x = \frac{142.56}{5.08} = 28$ cm

Answer: Marco's predicted foot length is 28 cm. (note that if you use a different method you will get a slightly different answer)

Suggested Strategies:

What you are looking for is a linear equation relating height and foot length. Once you've found one, you will use it to find foot length, knowing height.

There are a number of methods possible.

- Complete a scatter plot, drawing in the line of best fit and finding the equation of that line
- Use the median-median method
- Use the Mayer line method
- Enter the data into a graphing calculator to get the regression line

Additional Resources:

Visions Volume 1, Section 2.4, pp. 123-124
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Scatter Plots Activity A, Solving using trend lines
Khan Academy video: http://www.khanacademy.org/math/probability/regression/regression-correlation/v/fitting-a-line-to-data see fitting a line to data, estimating the line of best fit

Answer and Solution:

A) **This is the correct answer. These triangles are *not necessarily* congruent since they only have two congruent sides and no congruent angles indicated.**

B) These are congruent by ASA. Also, since two angles are congruent, then the third pair is too.

C) These are congruent by SSS.

D) These are congruent by ASA.

The answer is A.

Suggested Strategies:

Check all possible answers, and beside each one write the proof that confirms congruency. You should be left with only one that has no proof (meaning those triangles are *not* congruent).

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent
Khan Academy video:
http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)

QUESTION 51 3.1-MC-B

Answer and Solution:

The diagram identifies pairs of congruent angles and even though the congruent sides are not identified, the triangles share a side, which makes it congruent. Since the shared side is between pairs of congruent angles, the proof ASA is valid to prove congruency.

The answer is C.

Suggested Strategies:

When proving congruency (≅), first consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

Although this proof is perhaps not obvious since the congruent sides aren't identified, don't overlook the fact that the triangles share a side (AD).

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent
Khan Academy video:
http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)

Answer and Solution:

Although the top angle is not identified as being congruent in the two triangles in answer b, they must be since the other two pairs of corresponding angles are the same. Since the unidentified angles are now known to be congruent, two theories can be used to prove congruency – ASA and SAS.

A) There is not enough information, having two angles the same makes the triangle similar but not necessarily congruent

B) This may look like ASA but the congruent sides are not between the congruent angles so you can't conclude the triangles are congruent by that theory.

C) This is the correct answer.

D) This pair doesn't have corresponding sides that are congruent so you can't conclude they are congruent by ASA.

The answer is C.

Suggested Strategies:

Don't be discouraged if the correct answer doesn't jump out at you right away! Even though this is a multiple choice question, and you might expect to see the answer right away, there is often work or extra thinking needed to uncover the correct answer. Don't give up until you've tried all possibilities, in this case it involved a little extra thought.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent
Khan Academy video:
http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)

QUESTION 53 3.1-SA-A

Answer and Solution:

$\overline{MP} \cong \overline{NP}$ since P is the midpoint of MN, then MP and NP are congruent.

<LPM ≅ <OPN <LPM and <OPN are vertically opposite and therefore congruent.

<PML ≅ <PNO <PML and <PNO are alternate interior angles of a transversal through parallel lines, which means those angles are congruent.

With this information, we can say the triangles are necessarily congruent using the ASA theorem.

$\boldsymbol{\overline{MP} \cong \overline{PN}}$

<LPM ≅ <OPN

<PML ≅ <PNO

ΔLMP ≅ ΔONP by ASA

Suggested Strategies:

Make sure you fill in all the information that you know on your diagrams. This is important on all questions of a test, but especially on ones where there is obviously information that has been left out. Don't forget all of the angle relationships when filling in information – and seeing a transversal through parallel lines should remind you of those angle relationships.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent
Khan Academy video:
http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS)

Answer and Solution:

This diagram shows only two pairs of sides are congruent so you can eliminate ASA.

That leaves SSS and SAS. But you know that vertically opposite angles are necessarily congruent even if they aren't identified.

$$\overline{AC} \cong \overline{EC}$$

$$\angle ACB \cong \angle ECD$$

$$\overline{BC} \cong \overline{DC}$$

ΔABC ≅ ΔEDC by SAS

Suggested Strategies:

When proving congruency consider the three possible proofs (SSS, SAS, ASA). Starting with this will likely help you to eliminate one or two of the proofs as not having enough information pretty quickly.

In other cases, don't forget to go through the possible angle relationships for intersecting and transverse lines across parallel lines.

Additional Resources:

Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent
Khan Academy video: http://www.khanacademy.org/search?page_search_query=congruent+triangles
Congruent triangles (all conditions: SSS, ASA and SAS

Answer and Solution:

Step 1) Calculate the coordinates of P using the midpoint formula. They are (-16, 26).

$(x_m, y_m): \left(\frac{(-60+28)}{2}, \frac{(48+4)}{2}\right)$

$(x_m, y_m): \left(\frac{(-32)}{2}, \frac{(52)}{2}\right)$

$(x_m, y_m): (-16, 26)$

OR Calculate the distance between the two points and divide that distance by 2.

Step 2) Calculate the distance from M to P. It is 49.1935 m.

$$d = \sqrt{(-16 - -60)^2 + (26 - 48)^2}$$
$$d = \sqrt{(44)^2 + (-22)^2}$$
$$d = \sqrt{1936 + 484}$$
$$d = \sqrt{2420}$$
$$d = 49.1935 \text{ m}$$

Step 3) The missing angle (<MLP) is 70**°.** Using the Sine Law, calculate the missing measurements for **Δ**LMP. We know side MP is 49.1935 m, we can calculate side LM. It is 52.15 m. We can then calculate side LP. It is 13.55 m.

$$\frac{49.1935}{sin70} = \frac{x}{sin15} = \frac{y}{sin95}$$
$$x = \frac{49.1935(sin15)}{sin70}$$
$$x = 13.55\ metres\ (side\ LP)$$

$$y = \frac{49.1935(sin95)}{sin70}$$
$$y = 52.15\ m\ (side\ LM)$$

Step 4) Calculate the area of ΔLMP using either the Trig Formula or Hero's Formula. It is 332 m^2 (rounded to the nearest square metre).

Example of Trig Formula:

Area = $\frac{(13.55)(52.15)(sin70)}{2} = 332\ m^2$

OR Hero's Formula:

Half the perimeter: $\frac{49.19+13.55+52.15}{2} = 57.45$

$$Area = \sqrt{57.45(57.45 - 49.19)(57.45 - 13.55)(57.45 - 52.15)}$$
$$Area = \sqrt{57.45(8.26)(43.9)(5.3)}$$
$$Area = \sqrt{110410.52379}$$

Area = 332 metres^2

Step 5) Multiply the area of ΔLMP by 2, then multiply that by $5.

The total cost of painting is $3320.

(332m^2 x 2 x $5/m^2 = $3320)

You will charge ___$3320___ for painting the two triangles that make up the logo.

Suggested Strategies:

You need to find the area of the triangles and multiply that by $5.

What do you need to find the area of a triangle?

- The length of a base and altitude or
- The length of two sides and the angle between them (Trig area formula) or
- The length of all three sides (Hero's Formula)

Choose the method you think will work for you and find the measures you need.

You only need to do this once since the triangles are congruent.

Additional Resources:

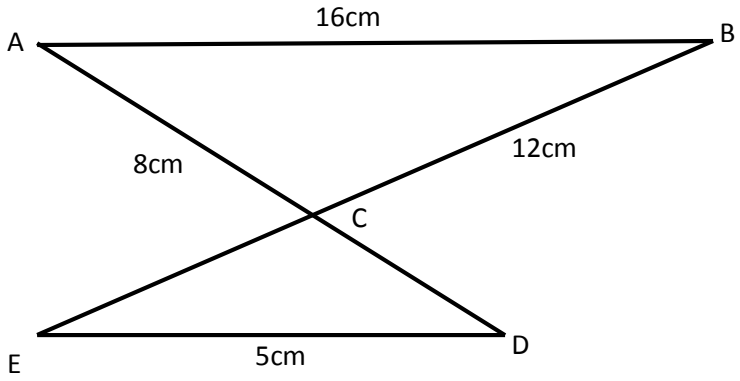
Visions Volume 1, Section 3.1, pp. 149-151, pp. 160-161

Khan Academy video: http://www.khanacademy.org/search?page_search_query=congruent+triangles Congruent triangles (all conditions: SSS, ASA and SAS

Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving triangles congruent

http://mathbits.com/MathBits/TISection/Trig/AreaTrigTri.htm

Answer and Solution:



$\Delta ABC \sim \Delta DEC$ because of the AA theorem – angle C is the same in both triangles, and because they are vertically opposite,
$\angle E \cong \angle B$ $(and \angle D \cong \angle A)$ because they are alternate interior angles.

In order to determine the total length of $\overline{BE}$, we need the length of $\overline{CE}$, so label the measure of $\overline{CE}$ as "x".

Since the triangles are similar, their sides must be proportional. Set up a proportion using corresponding sides:

$\frac{m\overline{CE}}{m\overline{BC}} = \frac{m\overline{ED}}{m\overline{AB}} \rightarrow \frac{x}{12} = \frac{5}{16}$

Cross multiply to determine the value of x

x = 12 x 5 ÷16 = 3.75 cm

To determine the length of side BE, add $m\overline{BC} + m\overline{CE}$

12 + 3.75 = 15.75cm

A) 2.5 cm is the measure of segment CD.

B) 3.75 cm is the measure of segment EC. The measure of segment BC must be added to this.

C) The result of adding the measures of segments CD and BC instead of EC and BC.

D) 15.75 cm is correct.

The answer is D.

Specific Strategies:

Some distractors might stand out: choices A) and B) are both fairly small, whereas C) and D) are both fairly large. An educated guess would eliminate A) and B), but we should remember that the drawings are never to scale.

- Label the figure with the given measurements
- Recognize that the triangles are similar because of AA

Alternate strategy:
Determine the scale factor k by dividing the lengths of corresponding sides: k = 3.2
Divide side BC by 3.2 to get the length of side CE = 3.75cm

- After completing the calculations, re-read the question and re-read the choices

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video:http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similarity-example-problems Similarity example problems

QUESTION 57 3.2-MC-B

Answer and Solution:

$m\angle y = 180° - 105° - 27° = 48°$ since the angles in every triangle add up to $180°$

A) Incorrect: This triangle only has one angle in common with triangle XYZ. Two side measures are given, but it is only possible to compare with one side of XYZ and we need at least two sides to prove SSS or SAS. (If you used the Sine Law to find the missing side of the original triangle, you will also see that the sides are not proportional.)

B) Incorrect: This triangle gives us three side measurements, but like option A, we would need to be able to compare at least two sides to prove SSS or SAS.

C) Correct: **This triangle is similar to XYZ. It has two angles in common because the third angle was calculated above to be 48°. So by AA, the triangles are similar.**

D) Incorrect: This triangle is not necessarily similar to XYZ because the information the triangle is not unique – you can make many triangles with those three features fixed.

The answer is C.

Specific Strategies:

- Keyword: Similar
- Recall theorems on similar triangles
- Determine the measure of the third angle in triangle XYZ
- Triangles are similar if they satisfy one of three theorems – AA, SSS or SAS

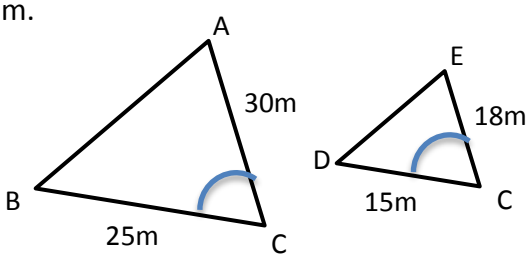
Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video:http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similarity-example-problems Similarity example problems

QUESTION 58 3.2-MC-C

Answer and Solution:

$\angle C \cong \angle C$ because they are vertically opposite angles in the original diagram.



Because two corresponding pairs of sides are given, as well as the angle contained, we should consider that the triangles are similar by SAS.

Check if the sides are proportional:

✓ $\frac{30}{18} = \frac{25}{15}$

A) We are only given one pair of corresponding angles and we don't know for sure that side $\overline{AB}$ is parallel to side $\overline{DE}$, so it's not AA.

B) Only 3 side lengths are given so we cannot consider SSS.

C) ASA is not a theorem for similarity.

D) **Because the proportions of two pairs of sides are the same and the angle contained by those sides is congruent, the triangles are similar by SAS.**

The answer is D.

Specific Strategies:

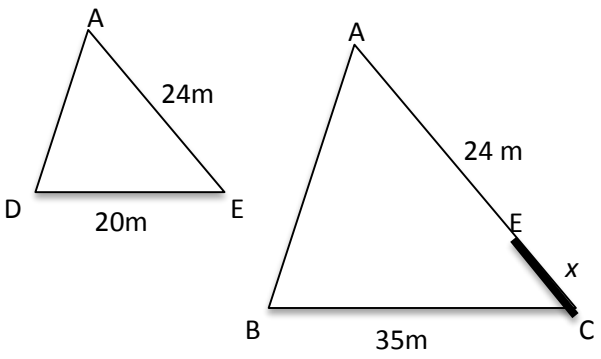
- Keyword: Similar
- Recall theorems on similar triangles
- Re-draw the triangles in the same orientation – *be careful with the rotation*
- Take note of other measurements which are not given in the problem
- Consider each of the similarity theorems

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video:http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similarity-example-problems Similarity example problems
Khan Academy video: http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similar-triangle-example-problems Similar triangle example problems

QUESTION 59 3.2-SA-A

Answer and Solution:



Because the unknown length is part of side $\overline{AC}$, it will be simpler to solve for the length of side $\overline{AC}$ first and then subtract 24m.

Set up a proportion using corresponding sides:

$$\frac{m\overline{DE}}{m\overline{BC}} = \frac{m\overline{AE}}{m\overline{AC}}$$

$$\frac{20}{35} = \frac{24}{m\overline{AC}}$$

$$m\overline{AC} = 42$$

If side AC measures 42 m, then the length of EC is:

$$42 - 24 = 18 \text{ m}$$

Answer: The length of segment EC is 18 m.

Specific Strategies:

- Keyword: Similar
- Label measurements directly on the figure

Highlight the length that the question is asking for

- Re-draw the triangles separately with the same orientation and include any measurements given
- Highlight corresponding sides with different colors
- Label the unknown measurement as "*x*"
- Reminder: re-do your calculations to double check, you will not get partial credit if your final answer is wrong!

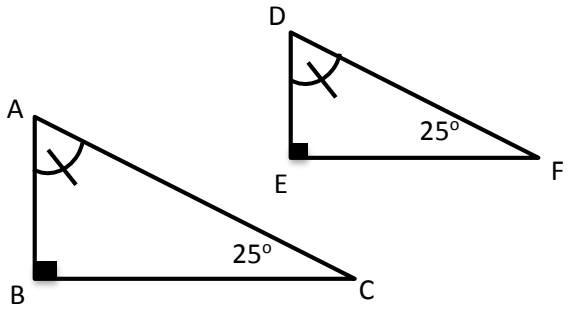
Additional Resources:

Visions Volume 1, section 3.2, p. 171 (Minimum Conditions for Similar Triangles)

Khan Academy video: http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similarity-example-problems Similarity example problems

QUESTION 60 3.2-SA-B

Answer and Solution:



$\text{m}\angle D = 180° - 90° - 25° = 65°$

Answer: The measure of $\angle D$ is $65°$.

Specific Strategies:

Label the given measurements on the figure

- Label the given measurements on the figure

- Draw the figures side by side in the same orientation

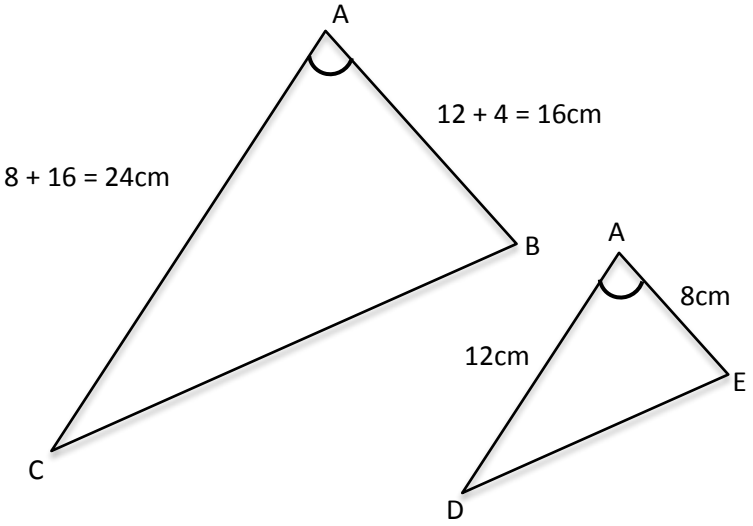
- Because the triangles have two angles in common, they must be similar by the AA theorem

- If two triangles have two angles in common, their third angle must also be common.

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 – Minimum Conditions for Similar Triangles
Khan Academy video: http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similar-triangle-basics Similar triangles basics

Answer and Solution:



Statement	Justification
The ratio of one pair of corresponding sides = 2	$\frac{m\overline{AC}}{m\overline{AD}} = \frac{24cm}{12cm} = 2$
$\angle A \cong \angle A$	Corresponding angles are congruent
The ratio of a second pair of corresponding sides = 2	$\frac{m\overline{AB}}{m\overline{AE}} = \frac{16cm}{8cm} = 2$
$\Delta ABC \sim \Delta AED$	SAS (Two triangles that have one congruent angle contained between corresponding sides of proportional length are similar)

☒ Yes, triangle ΔABC *is similar to* ΔAE

☐ No, triangle ΔABC *is not similar to* ΔAED

Specific Strategies:

- Label the figure with the given measurements
- Highlight the task

- Re-draw the triangles separately, with the same orientation
- Label the figure with the given measurements
- Label any other known measurements – *we know that angle A measures the same in both triangles*
- Highlight corresponding sides of the triangles
- Determine which geometric statement will allow you to prove the triangles are similar – SAS

Additional Resources:

Visions Volume 1, Section 3.2, p. 171 (Minimum Conditions for Similar Triangles)
Khan Academy video: http://www.khanacademy.org/math/geometry/similarity/triangle_similarlity/v/similarity-postulates Similarity postulates
Explore Learning Gizmos, http://www.explorelearning.com/ look up Similarity in Right Triangles

Answer and Solution:

Step 1: Pythagorean Theorem

$$(m\overline{AC})^2 = 60^2 + 80^2$$
$$m\overline{AC} = \sqrt{3600 + 6400}$$
$$m\overline{AC} = 100 \ m$$

Step 2: Apply Metric Relation

$$a \bullet b = c \bullet h$$
$$(m\overline{CB})(m\overline{AB}) = (m\overline{AC})(m\overline{BD})$$
$$60(80) = 100(m\overline{BD})$$
$$m\overline{BD} = \frac{4800}{100}$$
$$m\overline{BD} = 48 \ m$$

A) 36 m - this is $m\overline{AD}$

B) 48 m - correct

C) 64 m - this is $m\overline{DC}$

D) 69 m - this is the result from using an incorrect formula

The answer is B.

Suggested Strategies:

1) Put the measures onto the diagram.
2) If you are using formulas, make sure you label the triangle according to your formulas.
3) Identify the metric relation(s) that enable(s) you to solve for the unknown.
4) Consider that it might be more than one step.

Note: In this case you must apply Pythagorean theorem first before applying a metric relation formula.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles

Answer and Solution:

Step 1: Metric Relation

$$h^2 = m \cdot n$$
$$(m\overline{CD})^2 = (m\overline{AD}) \cdot (m\overline{DB})$$
$$m\overline{CD} = \sqrt{12(45)}$$
$$m\overline{CD} = \sqrt{540}$$
$$m\overline{CD} = 23.2379\ meters$$

Step 2: Area of Triangle ABC

$$\frac{m\overline{CD}(m\overline{AD} + m\overline{DB})}{2}$$

$$\frac{23.2379(12 + 45)}{2}$$

$$662m^2$$

The answer is C.

Suggested Strategies:

1) Orient the triangle in a way that is easiest for you.

2) Put the measures on the diagram and re-label if necessary.

3) Select the appropriate metric relation formula. *In this case we need 'h' and we are given 'm' and 'n'.*

4) Remember your basic area formulas. In this case we need area of triangle.

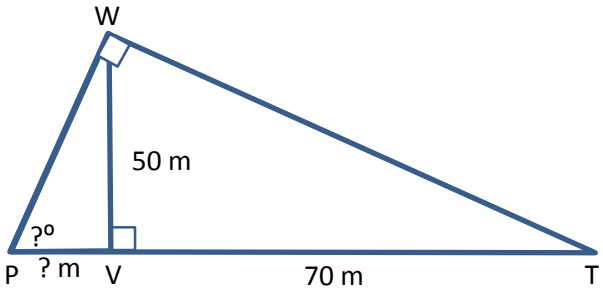
$$Area = \frac{base\ x\ height}{2}$$

Also be sure you get the area of the requested triangle. In this case the questions ask for triangle ABC – the largest of the three triangles.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles
http://www.youtube.com/watch?v=fdEBhf9SOYA

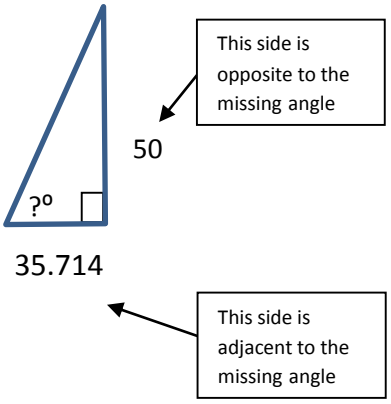
Answer and Solution:



Use Metric Relations to find the missing side.

$$h^2 = m \cdot n$$
$$50^2 = 70(m\overline{PV})$$
$$\frac{50^2}{70} = m\overline{PV}$$
$$\frac{250}{7} = m\overline{PV}$$
$$m\overline{PV} = 35.714$$

Use a Trigonometric Ratio to find the missing angle.



$\tan^{-1}\left(\frac{50}{35.714}\right)$=54.46°

The measure of angle WPV is $\approx 54.5°$.

Suggested Strategies:

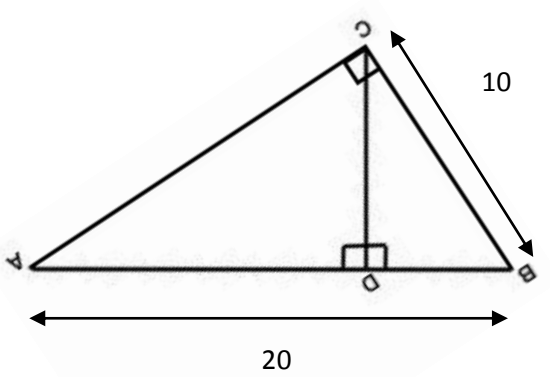
1) Redraw the diagram with only the triangles and measures (so the crane doesn't confuse you.) Now locate angle WPV.
2) If we find the measure of PV, we can use a trig ratio (tangent) to find the missing angle.
3) To find the missing measure we'll use a metric relation that uses the measures we have.

Make sure your calculator is set to degree mode.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles

Answer and Solution:



Step 1: Solve for $m\overline{DB}$

$$a^2 = n \cdot c$$
$$(m\overline{BC})^2 = (m\overline{BA})(m\overline{DB})$$
$$10^2 = 20(m\overline{DB})$$
$$\frac{100}{20} = (m\overline{DB})$$
$$(m\overline{DB}) = 5$$

Step 2: Solve for $(m\overline{AD})$

$$(m\overline{AB}) = m\overline{AD} + m\overline{DB}$$
$$20 = m\overline{AD} + 5$$
$$\boldsymbol{m\overline{AD} = 15}$$

The measure of $\overline{AD}$ is 15 m.

Suggested Strategies:

1) If necessary, orient the triangle in a way that is easiest for you.

2) If necessary, re-label the vertices of triangle according to your memory aid.

3) Add the numerical information to the diagram.

4) Identify the metric relation formula that enables you to solve for the unknown or set up a proportion knowing the three triangles are similar:

$$\frac{hyp_{small}}{short\ side_{small}} = \frac{hyp_{large}}{short\ side_{large}}$$

$$\frac{10}{\overline{BD}} = \frac{20}{10}$$

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles

Answer and Solution:

Step 1: Solve m$\overline{PF}$

Distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

P(15, 60) and F(60, 90) $\sqrt{(60 - 15)^2 + (90 - 60)^2} = m\overline{PF}$

$$m\overline{PF} = \sqrt{45^2 + 30^2} = 54.08$$

Step 2: Solve $m\overline{ZP}$

Set-up equation of line ZF $y = ax + b$

Given P(15, 60) and F(60, 90)

$$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 60}{60 - 15} = \frac{30}{45} = \frac{2}{3}$$

$y = \frac{2}{3}x + b$ *Now we have to solve for 'b'.*

Solve for 'b' by substituting the coordinates of a point on the line into the equation. In this case we have a choice between point P and F. It does not matter which one you choose.

Using the coordinates of point P we have,

$60 = \frac{2}{3}(15) + b$

$60 = 10 + b$

$b = 50$ ← This is the *y*-coordinate of point Z

Step 3: Solve $m\overline{ZP}$

Distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Z(0, 50) and P(15, 60)

$\sqrt{(15 - 0)^2 + (60 - 50)^2} = m\overline{PZ}$

$$m\overline{PZ} = \sqrt{15^2 + 10^2} = 18.03$$

Step 4: Metric Relations to solve m$\overline{PK}$

$$h^2 = m \cdot n$$
$$(m\overline{PK})^2 = (m\overline{ZP})(m\overline{PF})$$
$$m\overline{PK} = \sqrt{18.03(54.08)}$$
$$m\overline{PK} = 31.23$$

Step 5: Solving for m$\overline{ZK}$

$$\mathrm{m}\overline{ZK} = \sqrt{(m\overline{ZP})^2 + (m\overline{PK})^2}$$
$$\mathrm{m}\overline{ZK} = \sqrt{18.03^2 + 31.23^2}$$
$$\mathrm{m}\overline{ZK} = 36.06$$

Step 6: Sum up the lengths of three line segments

$$m\overline{ZK} + m\overline{PK} + m\overline{PF} = 121.37\ meters$$

The total combined distance is 121.37 m.

Suggested Strategies:

1) We know we have to find distances of three line segments. Line segment $\overline{PF}$ can be found with the distance formula since we are given the coordinates of point P and point F.

2) We know Point Z is on the *y*-axis. This means the *x*-coordinate is zero. To solve for the *y*-coordinate we can set up an equation of a line.

$y = ax + b$

Recall: 'a' in the equation represents the slope $\frac{y_2 - y_1}{x_2 - x_1}$ and '*b*' represent the *y*-intercept.

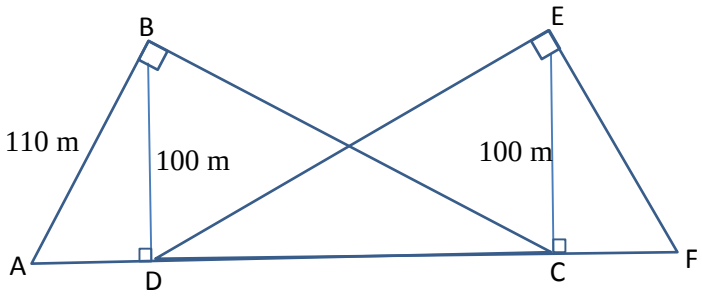
Additional Resources:

Visions Volume 1, Section 3.3, p. 181

Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles

Answer and Solution:

Here is an example of an appropriate method



Step 1: We can prove that $\Delta BCD \text{ and } \Delta ECD$ are congruent by theorem SAS:

Justification:

- $m\overline{CD}$ is a shared length to both triangles
- $m\overline{BD}$=m$\overline{CE}$ Given information
- $m\angle$ABC and $m\angle$DEF are both 90°

$m\overline{BC}$=m$\overline{DE}$ since they are corresponding sides of congruent triangles.

Step 2: Apply Pythagorean Theorem to find m$\overline{AD}$

$$m\overline{AD} = \sqrt{110^2 - 100^2}$$
$$m\overline{AD} = 45.8258$$

Step 3: Apply Metric Relation to find m$\overline{DC}$

$$h^2 = m \cdot n$$
$$(m\overline{BD})^2 = (m\overline{AD})(m\overline{CD})$$
$$100^2 = 45.8258(m\overline{CD})$$
$$\frac{100^2}{45.8258} = (m\overline{CD})$$

$$(m\overline{CD}) = 218.2177$$

Step 4: Apply Pythagorean Theorem to find $m\overline{BC}$

$$(m\overline{BC}) = \sqrt{218.2177^2 + 100^2}$$
$$(m\overline{BC}) = 240 \text{ metres}$$

The length of the cable represented by $\overline{DE}$ is indeed 240 m.

Suggested Strategies

1) Put the numerical information into the diagram.
2) Since triangle DEF has no information about it and triangle DEC only has one measure, convince yourself that the triangles with more measures can be used.
3) See that $\overline{BC}$ is the same length as $\overline{ED}$ so if you find $m\overline{BC}$ you'll know $m\overline{DE}$.
4) Use Pythagoras to find $m\overline{AD}$.
5) Use a metric relation to find $m\overline{DC}$.
6) Use Pythagoras again to find $m\overline{BC}$.
7) $\overline{BC}$ is the same length as $\overline{DE}$.

Additional Resources:

Visions Volume 1, Section 3.3, p. 181
Explore Learning Gizmos, http://www.explorelearning.com/ look up: similarity in right triangles

Answer and Solution:

x (number of hours)	*f(x)* (total number of cells)
0	50
1	50 x 2 = 100
2	100 x 2 = 200
3	200 x 2 = 400

An initial amount which increases by the same multiplier is an exponential function

$$f(x) = a(c^x)$$

A) $f(x) = 50(x^2)$
Since we see the term x^2, this is a quadratic or second-degree polynomial function.

B) **$f(x) = 50(2^x)$**
This is an exponential function with an initial value of 50 and a base of 2, which means that it is multiplying by 2 for each increase in *x*.

C) $f(x) = 50 + 2x$
This is a linear function or first-degree polynomial function since we see no exponents.

D) $f(x) = 2(50^x)$
This is an exponential function, but the initial value and the base are switched.

The answer is B.

Specific Strategies:

- Since the choices are different function rules; the goal of the problem is to translate the situation into a functional model.
- Make a table of values to get a clearer picture of the relation
- Doubling the number of cells means you must multiply by 2 for each hour
- Look on your memory aid for the rule of an exponential function

Additional Resources:

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Exponential functions – Activity A

Answer and Solution:

A) This option is a step function, which means that the amount of rain accumulated remains constant for a period of time and jumps abruptly at critical values; it does not make sense to represent a constant increase of 5mm per hour

B) This option is a linear function of first-degree polynomial function. Its initial value is 120 and it has a positive slope, indicating a constant increase

C) This option does not illustrate a constant increase; the amount of rain increases sharply at first, then accumulates more slowly

D) This option appears to be an exponential function, which would mean that the amount of rain increases by some multiplying factor instead of a constant rate

The answer is B.

Specific Strategies:

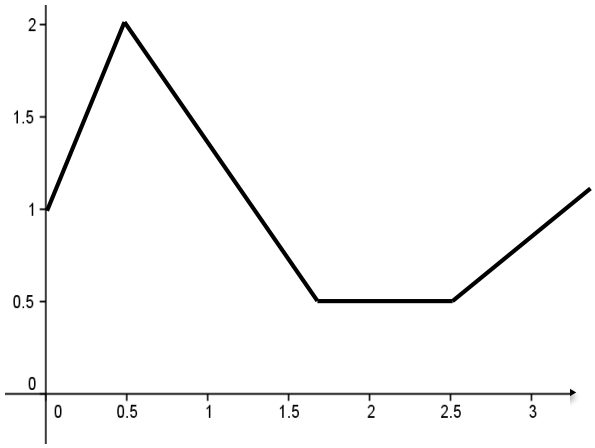
Since the choices are graphs, this is a problem that can be answered by observation; no calculations need to be done.

- Consider all the functional models on your memory aid
- The linear function model is the only one which offers an interpretation for a constant increase over time

Additional Resources:

Visions Volume 1, Section 1.2, p. 22 (Lines in the Cartesian Plane)
Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Khan Academy video: http://www.khanacademy.org/math/algebra/algebra-functions/relationships_functions/v/basic-linear-function Ex. Constructing a Function

Answer and Solution:



A) **TRUE! The function has no zeros**

B) FALSE- The domain is $[0, 3]$

C) FALSE- The function is positive from [0, 0.5]

D) FALSE- The function has a maximum at (0.5,2)

The answer is A.

Specific Strategies:

Since the answers are properties of functions, we need to analyze the graph to determine the answer. No calculations are necessary.

- Find the definitions for properties of functions on your memory aid
- Make sure you know which properties concern *x* values and which properties concern *y*-values

***x-values**: domain, x-intercept intervals of increase and decrease, positive and negative intervals*

***y-values**:*
range, y-intercept, extrema (maximum and minimum)

Zeros:
A value for x that produces a value of 0 for y
Or the x-value when line passes x-axis

Extrema:
A point which has the highest or lowest value of the function.

Additional Resources:

Visions Volume 2, Section 4, (Revision), pp. 7-8 (Properties of Functions)
Khan Academy video: Comparing features of functions 2 (example 1) ; Interpreting features of functions 2 (example 1) ; When a function is positive or negative
Khan Academy video: http://www.khanacademy.org/math/trigonometry/functions_and_graphs/analyzing_functions/v/when-a-function-is-positive-or-negative

QUESTION 71 4.1-SA-A

Answer and Solution:

A) The x only has a coefficient so this is a linear function and would be a line.

B) **Correct - The x is an exponent so this is an exponential function and would curve up (or down) sharply and cross the y-axis.**

C) The x is squared so this is a quadratic function and would be a U-shape.

D) The x is a denominator so this is an inverse (rational) function and, in this particular case, would not cross the y-axis (since $x \neq 0$).

The answer is B.

Suggested Strategies:

- Compare the graph of the functions studied and their corresponding rules, to the ones provided in the question.

Additional Resources:

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Visions Volume 2, Section 4.3, p. 39 (Exponential Functions)
Khan Academy video; http://www.khanacademy.org/math/trigonometry/exponential_and_logarithmic_func/exp_growth_decay/v/exponential-growth-functions – Exponential grown functions

QUESTION 72 4.1-SA-B

Answer and Solution:

The range is the set of all possible values of *y* from least to greatest (bottom to top).

The lowest value on the graph is *y* = 0 and the highest value on the graph is y= 25.

Therefore the range is]0, 25].

The range of the function is]0, 25]. It means the ball will reach its highest point at 25 m and its lowest point at 0 m, or in other words, it will hit the ground.

Specific Strategies:

Look for the lowest value of *y* on the graph and the highest value of *y* on the graph.

Use interval notation in providing your answer. (Remember that the smallest value comes first.)

Additional Resources:

Visions Volume 2, Section 4 (Revision), p. 7 (Properties of Functions)

Answer and Solution:

Determine the y-value when x = 8 for the function g(x)

$$g(8) = 10(8)^2$$

BEDMAS: Remember to apply the exponent first

$$g(8) = 10 \times 64$$
$$= 640$$

So, the step function begins at a y-value of $640 after 8 hours of work. The price doesn't go up until 12 hours.

x (Number of hours)	*y* (Cost $)
[8, 12[	$640
[12, 16[	$640 + 250 = $890
[16, 20[	$890 + 250 = $1140
[20, 24[	$1140 + 250 = 1390
[24, 28[	**$1390 + 250 = $1640**

For $1640, it will take 24 to 28 hours, not including 28 hours.

That job would have taken between 24 and 28 hours.

Specific Strategies:

- The first unknown that needs to be determined is the cost for 8 hours of work
- Make sure to keep your work organized from this point – a table is a great idea
- According to the step function, the open circle at *x* = 12 is a critical value, so the cost will "jump" starting at 12 hours

Additional Resources:

Visions Volume 2, Section 4.1, p. 17 (Families of Functions)
Visions Volume 2, Section 4.2, p. 28 (Second-degree Polynomial Function)
Visions Volume 2, Section 4.3, p. 53 (Piecewise Function)
Step Graphs: http://www.youtube.com/watch?v=LUshzsvoGZU

Answer and Solution:

A) **This parabola opens upward, so it must have a positive "a" value. Also, when $x = 1$ we can check the y-value by using the rule: $f(x) = 2(1)^2 = 2$. Since the parabola appears to pass through the point (1, 2), we can conclude that this is the correct graph.**

B) This parabola opens upward, so it must have a positive "a". However, when $x = 1$, the value of the function is clearly less than 1, which does not satisfy the equation $f(x) = 2x^2$**.**

C) This parabola opens downward, so parameter "a" is a negative value.

D) This parabola opens downward, so parameter "a" is a negative value.

The answer is A.

Specific Strategies:

Remember:
For a quadratic or second-degree polynomial function, $y = ax^2$

Since parameter "a" is positive, the parabola should open upward.

Since parameter "a" is longer than 1, the parabola should be narrower than the basic function $f(x) = x^2$.

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

QUESTION 75 4.2-MC-B

Answer and Solution:

Just by looking… only B matches the graph

A) quadratic opening up

B) quadratic opening down

C) exponential function

D) linear function

Confirm by testing the point…

A) $-4 = 0.25(4)^2$
$-4 = 4\ false$

B) $\mathbf{-4 = -0.25(4)^2}$
$\mathbf{-4 = -4}\ \boldsymbol{true}$

C) $-4 = -0.25^4$
$-4 = 0.0039\ false$

D) $-4 = 0.25(4) + 1$
$-4 = 2\ false$

Since only one statement is true, that must be the answer.

The answer is B (a quadratic with a negative parameter "a").

Suggested Strategies:

Imagine the shape of each type of function.

- A quadratic (ax^2) looks like a U.
- An exponential (ac^x) is a curve that either increases or decreases over the whole domain.
- A linear ($ax + b$) is a straight line.

Recall for a quadratic function if:

- Parameter "a" is positive the parabola opens upward → U is up- Happy Face!
- Parameter "a" is negative the parabola opens downward → U is down- Sad Face!

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving graphing _quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

Answer and Solution:

A) $f(x) = -3x^2$
This function opens down (decreases after the vertex) but the table of values increases after the vertex

B) $f(x) = -0.3x^2$
This function opens down (decreases after the vertex) but the table of values increases after the vertex

C) $\boldsymbol{f(x) = 0.3x^2}$
This function opens up and when a non-zero x-value is tested, it gives the corresponding value for f(x):
$f(5) = 0.3(5)^2 = 0.3 \times 25 = 7.5$

D) $f(x) = 3x^2$
This function opens up but when a non-zero x-value is tested, it doesn't give the corresponding f(x).
$f(5) = 3(5)^2 = 3 \times 25 = 75 \neq 7.5$

OR

Calculate parameter "a" algebraically using one of the points given:

$$f(x) = ax^2$$

for example: (5, 7.5)

$$7.5 = a(5)^2$$
$$\frac{7.5}{25} = \frac{a \times 25}{25}$$
$$0.3 = a$$

The answer is C.

Suggested Strategies:

- See what patterns you notice among the data.
  - Symmetry around (0,0)
  - (0, 0) is a minimum so the "a" value will be positive.
  - Since you are told it is quadratic, you know it's in the form ax^2 so you can calculate a.

Test the two positive functions to see which one works.

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video: https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

QUESTION 77 4.2-SA-A

Answer and Solution:

The rule for a second degree function is the form $f(x) = ax^2$

Use the coordinates (5, 10) in the function form to determine parameter "a".

$$f(x) = ax^2$$
$$(10) = a(5)^2$$
$$10 = 25a$$
$$\frac{10}{25} = \frac{25a}{25}$$
$$\frac{2}{5} \text{ or } 0.4 = a$$
$$f(x) = 0.4x^2$$

or $f(x) = 0.4x^2$

The rule of the function is $f(x) = 0.4x^2$

Specific Strategies:

Recall the rule for the second degree function with vertex at the origin is $y = ax^2$.

Remember that in any algebraic equation, if you substitute known values, you can solve for the unknown remaining.

ASK YOURSELF:
What information is given in the graph?
An (x, y) point is given in the graph.
If you substitute x & y, then only parameter "a" will remain to be determined.

Additional Resources:

Visions Volume 2, pp. 28-29
Explore Learning Gizmos, http://www.explorelearning.com/ look up:

- Quadratic Functions
- Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

Answer and Solution:

Function	Graph
E) $y = 5x^2$	***g***
F) $y = -0.2x^2$	***h***
G) $y = x^2$	***f***
H) $y = -x^2$	***k***

Suggested Strategies:

- Remember what effect parameter "a" has on the curve:
  - Positive opens up
  - Negative opens down
  - The larger the absolute value of "a" the narrower the curve

Additional Resources:

Visions Volume 2, Section 4.2, pp. 28-29
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

Answer and Solution:

Company A:

You can use any of the points to substitute them in ***y = ax²***

$y = ax^2$

$(1800) = a(10)^2$

$1800 = 100a$

$\frac{1800}{100} = \frac{100a}{100}$

$a = 18$

$rule: y = 18x^2$

Let *x* be the length of side

Let *y* be the cost of turf

Now that you have the rule, you can substitute accordingly:

Turf piece of 22.5m (*x* value):
$y=18 \times (22.5)^2$
$y=9112.5$
$9 112.50

Company B:

Calculate the area of the square piece of turf:

$$22.5 \times 22.5 = 506.25\ m^2$$

Reading the graph, you can see that 506.25 m^2 corresponds to a cost of **$11 000**.

Gordon will buy the turf at the lowest price and therefore he will buy from Company A.

Gordon will pay $9 112.50 for the turf.

Suggested Strategies:

Company A:

Turf is sold in SQUARE pieces and the cost depends on the AREA (s^2)

ASK YOURSELF:
Which kind of function is associated with squaring a number? ***y = ax²***

Substitute (*x*, *y*) value and solve for "a". This will give you your rule and you can then substitute and solve for *y*.

Company B:

Before you can use this graph, you need to calculate the area of the turf Gordon needs.

Additional Resources:

Visions Volume 2, pp. 28-29
Khan Academy video:https://www.khanacademy.org/math/algebra/quadratics/solving_graphing_quadratics/v/graphing-a-quadratic-function
http://www.purplemath.com/modules/grphquad.htm
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Quadratics in Polynomial Form- Activity A and Activity B (for these, you must keep the b and c sliders at position 0)

Answer and Solution:

Exponential Function:
$f(x) = a(c)^x$
where: a is the initial value and
c is the base

Given the value increases, $C > 1$

2/100 = 0.02 and then adding it to 1

C = 100% + 2% = 102% = 1.02

The correct equation is $f(x) = 275\,000\,(1.02)^x$

A) $f(x) = 275\,000\,(0.02)^x$ This would be a decreasing function since the base is less than 1.

B) $f(x) = 275\,000\,(1.02)^x$ This is correct.

C) $f(x) = 275\,000\,(1.2)^x$ m This would represent a growth rate of 20% not 2%.

D) $f(x) = 275\,000(0.98)^x$ This would represent a decay of 2% (decreasing).

The answer is B.

Suggested Strategies:

- Find the type of function for the situation.
- Calculate "c" by using the percentage.
- Remember that in growth situations we add to 100%.

Additional Resources:

Visions Volume 1, pp. 39-41

QUESTION 81 4.3-MC-B

Answer and Solution:

The initial value is 505, not 0.94.

The bike's value decreases by 6%, not 94%.

This is a decreasing function because 0.94 is less than 1.

The value of the bike will be \$272 in the year 2020.
Proof:
2020 – 2010 = 10 years
$f(10) = 505(0.94)^{10} = 272$

A) The initial value is 0.94.
FALSE – the initial value is \$505.

B) The bike decreases by 94% yearly.
FALSE – the rate at which it decreases is 1 – 0.94 = 0.06, or 6%

C) The graph is an increasing function.
FALSE – since the base is less than one, it is a decreasing function.

D) In the year 2020, the value of the bike will be \$272. TRUE!

2000 – 1990 = 10 years
$f(10) = 505(0.94)^{10} = 272$

The answer is D.

Suggested Strategies:

- Determine what the initial value is and compare with A).
- Determine the percentage decrease and compare with B).
- Determine if the function is increasing and compare with C).
- Calculate the number of years from 2010 to 2020 and replace value in *x*. Compare with D).

Additional Resources:

Visions Volume 1, pp. 39-41

QUESTION 82 4.3-SA-A

Answer and Solution:

$f(x) = ac^x$

c = 1 – 0.35 = 0.65

f(x) = a $(0.65)^x$

10.21 = a $(0.65)^5$

$\frac{10.21}{0.1160} = \frac{0.1160\ a}{0.1160}$

a = $88.02

Answer: The initial price of the video game is $88.02.

Suggested Strategies:

- Notice that this is an exponential function that is decreasing.
- Write the equation for this function.
- Calculate the value of "c" which uses the percentage.
- Fill in x with the number of years and y with the price after 5 years.
- Work backwards to find *a,* the initial price.
- Remember that in decay or decreasing situations we subtract from 100%.

Additional Resources:

Visions Volume 1, pp. 39-41

QUESTION 83 4.3-SA-B

Answer and Solution:

$137\,858 = 4500\,(1.33)^x$

OR

x	$4500(1.33)^x$	*f(x)*
1	$4500(1.33)^1$	5985.0
2	$4500(1.33)^2$	7960.1
3	$4500(1.33)^3$	10586.9
4	$4500(1.33)^4$	14080.5
5	$4500(1.33)^5$	18727.1
6	$4500(1.33)^6$	24907.1
7	$4500(1.33)^7$	33126.4
8	$4500(1.33)^8$	44058.1
9	$4500(1.33)^9$	58597.3
10	$4500(1.33)^{10}$	77934.4
11	$4500(1.33)^{11}$	103652.7
12	$\mathbf{4500(1.33)^{12}}$	**137858.1**
13	$4500(1.33)^{13}$	183351.2

x = 12 years later

2005 + 12 = 2017

Answer: In 2017 the number of bacteria will be 137 858.

Suggested Strategies:

- Replace *y* by the number of bacteria given.
- Work backwards to find the value of *x* by guess and check.

OR

- Set up a table of values for the function and find the *y* value you are looking for.
- In the table, you can jump ahead to where you think *x* would work.
- Your table should contain more than two calculations in order to show evidence of your thinking.

Additional Resources:

Visions Volume 1, pp. 39-41

QUESTION 84 4.3-SA-C

Answer and Solution:

$f(x) = ac^x$
$a = initial\ value = 5000$ (principal)
$c = growth\ rate = 1 + 0.025 = 1.025$

OR 100% + 2.5% = 102.5% or 1.025

$f(x) = 5000\ (1.025)^x$
where:
x is the number of years and
$f(x)$ is the total of principal and interest

$f(10) = 5000\ (1.025)^{10}$

$f(x) = \$6400.42$

6400.42 – 5000 = $1400.42

Answer: Sophia will have made $1 400.42 profit in 10 years.

Suggested Strategies:

- Notice this is an exponential function and that it is increasing so the base is greater than 1.
- Write the equation associated with the function.
- Substitute the initial value for "*a*".
- Determine "*c*" by using the percentage.
- Recognize that *x* represents the number of years and *y* represents the total amount.
- Plug in number of years for "*x*".

Additional Resources:

Visions Volume 1, pp. 39-41

Answer and Solution:

Town A: constant function

$f(x) = 5000$,

where:

x is the number of years elapsed since 1960

$f(x)$ is the total inhabitants.

Town B: exponential function

$2020 - 2001 = 19$ years elapsed.

$f(19) = 2000\ (1.022)^{19} = 3024$ inhabitants

Town C: linear function

$2020 - 2010 = 10$ years elapsed

$f(x) = 5000 - 50\ (x)$

$f(19) = 5000 - 50\ (19) = 4050$ inhabitants.

Town D : exponential function

For $c = 100\% + 5\%$

$= 105\% = 1.05$

$a =$ initial population of 1500

$f(x) = ac^x$

$f(x) = 1500\ (1.05)^x$

$x = 2020 - 2006 = 14$

$f(14) = 1500\ (1.05)^{14} = 2969$ inhabitants

Total:

5000 + 3024 + 4050 + 2969 = 15 043 inhabitants

Answer: The population of the new city will be 15 043.

Suggested Strategies:

- Determine the type of function for each town and write the equation.
- Determine the number of years that pass between the time of the merging and the creation of each town.
- Replace x by the number of years that pass in each equation.

Additional Resources:

Visions Volume 1, pp. 39-41

QUESTION 88 4.4-MC-B

Answer and Solution:

Read each statement carefully.

1. A customer that spends $150 will receive a $10 discount.

   False-$150=$15 discount

2. A customer that spends $75 will receive a $5 discount.

   True – In values between the endpoints of the step has the same y-value.

3. A customer will receive a $5 discount when spending less than $100.

   False – Be careful with the less than $100. Less than $100 includes less than $50. Less than $50 equals $0 discount.

4. A customer will receive twice as much of a discount when spending $200 than $100.

   True – The discount for $200 = $20 and $100= $10

5. A customer will receive no discount when spending less than $50.

   True – Less than $50 is the step on the x-axis.

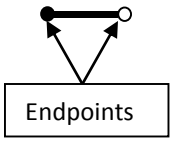
The answer is A.

Suggested Strategies:

1) "*greatest integer function*" is another name for *step function*.

2) Read each statement carefully and determine if it is true or false.

3) When reading the endpoints of the steps know the difference between an open circle and filled in circle.



- A closed point means it is included.
- An open point means it is not included.

4) The question is looking for the <u>*true*</u> statements. Sometimes the question wants the *false* statements. Take a moment to highlight the word <u>*true*</u> and indicate each statement as true or false as you read them.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53

QUESTION 89 4.4-SA-B

Answer and Solution:

Making a table to show how the cost changes over time.

ANSWER

The question asks for 5 hours which is located in the interval $]4.5,5.5]$ at a cost of $65.

Cost to Park a Car

Time (hours)	Total Cost
$]0,0.5]$	$40
$]0.5,1.5]$	$45
$]1.5,2.5]$	$50
$]2.5,3.5]$	$55
$]3.5,4.5]$	$60
$]4.5,5.5]$	$65
$]5.5,6.5]$	$70

Notice the use of intervals. The interval $]0, 0.5]$ indicates the hours ranging from anything above 0 up to and including 0.5 hours. Although the customer parks a car for ¼ of an hour, they will be charged $40.

OR

You could also consider the very specific case: $40 for the first half hour plus 4.5 hours –> rounded up to 5 hours x $5 per hour: 40 + (5 x 5) = 40 + 25 = 65.

Answer: The customer will pay $65 for parking 5 hours.

Suggested Strategies:

1) Identify that this problem is a step function.

2) Note the key words in the problem.
"for each additional hour or <u>part thereof.</u>

Take a moment to highlight the key words.

"part thereof" means that the customer will be charged for $5 even if they did not complete one hour.

3) Recall the use of intervals.
Notice the difference in the brackets.
$]0, 0.5]$

The bracket here means including 0.5, while the other bracket does not include 0.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53

QUESTION 90 4.4-SA-C

Answer and Solution:

Company A:

$$\frac{100\ gb}{\$20} = \frac{200\ gb}{\$x} \rightarrow x = \$40$$

OR

We can see the line passing through the origin (0, 0) and the given point (100, 20)

$$\$20\ per\ 100\ gigabytes$$
$$\$0.20\ per\ 1\ gigabyte$$

The function rule to calculate the cost in relation to the internet usage (gigabyte) will be $f(x) = 0.20x$

The cost of 200 gigabyte $f(200) = \$40$

Company B:

From the graph – f(200) = $25
(Note that it isn't $35 because it's the solid dot that indicates the y-value.)

The difference: $\$40 - \$25 = \$15$

Answer: The difference in cost is $15.

Suggested Strategies:

1) Since we can't read off the graph for the cost of 200 gb from Company A, we need to look further.
   - The text says each 100 gb costs $20 so we could set up a proportion or
   - The graph shows the same information but we can find a unit rate which will be the "*a*" in *y* = *ax* + *b*. *b* will be 0 since the line goes through the origin.

Recall: '*a*' in the equation represents the slope $\frac{y_2 - y_1}{x_2 - x_1}$ and '*b*' represent the *y*-intercept. (In this case it is 0 since the line passes through the origin.

2) By looking at the graph we can see the exact value of the cost of Company B at 200 gb. Be careful not to take the $35 value. Remember the difference between a white circle and a black circle.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53

Answer and Solution:

Step 1: Determine the period of the function

5 complete cycles = 400 seconds 1 complete cycle = 80 seconds

Step 2: Determine how many complete cycles from 8:00-8:15 AM

Since our unit of time on our graph is in seconds we need to convert minutes to seconds.

15 minutes x 60 seconds/minute = 900 seconds

Determine the number of complete cycles in 900 seconds

$$\frac{900\ seconds}{80\frac{seconds}{cycle}} = 11.25\ cycles$$

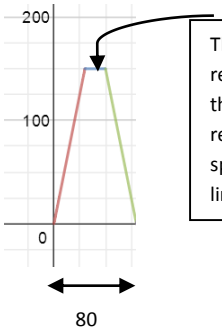
11 Full Cycles means that the ball will return back to where it started (i.e. ground)
0.25 of a cycle represents quarter of a full cycle. (0.25 x 80seconds = 20 seconds)

OR

The graph (5 cycles) covers 400 seconds, so 800 seconds covers the graph twice, leaving 100 seconds (900 – 800) left over to being the third time. From reading the graph, you can see that at 100 seconds, the ball is 100 cm above the ground. HOWEVER, even though it *looks* like 100, make sure by showing the following:

Step 3: Determine the height the ball is relative to the ground at 20 seconds

To find the exact value we need to break a cycle into pieces.



The constant piece represents 20 seconds of the cycle. That means the remaining 60 seconds is split between the other two lines.

At 20 seconds the ball will be moving upwards. To know the exact height we will have to find the equation of a line: $y = ax + b$.

We have two points on the line. $P_1(0,0)$ and $P_2(25,125)$

$a = \frac{y_2 - y_1}{x_2 - x_1} = \frac{125-0}{25-0} = \frac{125}{25} = 5$ and $b = 0$ since the *y*-intercept is at the origin.

The equation of the line is $y = 5x$

To find the height of the ball at 20 seconds, substitute x=20 and solve for y.

$$y = 5(20) = 100\ cm$$

The ball is <u>100</u> cm off the ground when the mascot stops moving.

Suggested Strategies:

1) Determine the period (the length of a full cycle) of the periodic function.

Note: The scale of x-axis is 100/4 = 25 seconds per grid mark.

In this case you cannot determine the exact value of the period from looking at one cycle on the graph. Instead we can see that 5 full cycles equals 400 seconds.
(1 cycle = 80seconds)

2) At 8:00 AM the ball starts at ground level and moves for 15 minutes. We need to figure out how many complete cycles we have completed in 15 minutes and see what's left over.

3) Write the equation of a line given two points.
$$y = ax + b$$

Recall: 'a' in the equation represents the slope $\frac{y_2 - y_1}{x_2 - x_1}$ and 'b' represent the *y*-intercept.

Additional Resources:

Visions Volume 2, Section 4.4, p. 53

Answer and Solution:

Step 1: Solving for the parameter "*a*" in the second-degree function

Note: The second-degree function is connected to the constant function. To determine the coordinates of the point connecting the two functions we need to look at the interval in the function rule.

When *x* = 80, the *y*-coordinate is at 256.

Substitute the coordinates into the second-degree equation to solve for '*a*'

$$a(80)^2 = 256$$
$$a = \frac{256}{6400} = \frac{1}{25} \; or \; 0.04$$

Step 2: Solving for parameter "*b*" in the linear equation.

The function connecting the linear function is the constant function.

When *x* = 160, *y* = 256

$$256 = -1.25(160) + b$$
$$256 + 200 = b$$
$$b = 456$$

Step 3: Find the x coordinates when y= 144

In $f(x) = 0.04x^2$

$144 = 0.04x^2$

$3600 = x^2$

$x = 60$

In $f(x) = -1.25x + 456$

$144 = -1.25x + 456$

$-312 = -1.25x$

$x = 249.6$

Step 4: The length of tape

249.6 cm — 60 cm = 189.6 cm

The length of the piece of reflective tape is 189.6 cm

Suggested Strategies:

1) Identify the type of functions in the piecewise function.

   Remember a piecewise function is made up of different functions defined by a certain interval of *x*-values (domain).

2) To solve for the missing parameters you will to know the coordinates of the points that connect the functions together

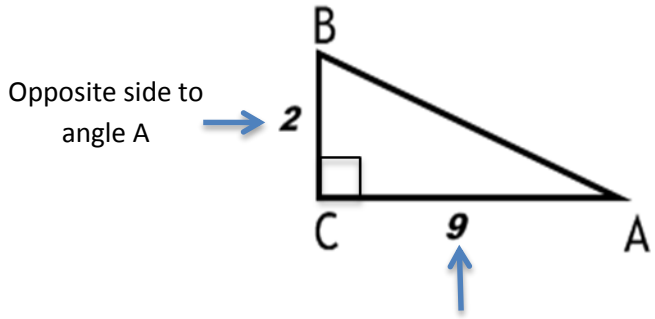
3) Find the x coordinates of the two extremities of the tape

Additional Resources:

Visions Volume 2 Section 4.4 p. 53
http://www.mathsisfun.com/sets/functions-piecewise.html
http://www.purplemath.com/modules/strtlneq.htm

QUESTION 93 5.1-MC-A

Answer and Solution:



Since the opposite and adjacent sides are given, the correct trigonometric ratio to select is Tangent (tan).

Tangent is the ratio of $\frac{\text{opposite side of an angle}}{\text{adjacent side of an angle}}$

So, $\tan A = \frac{2}{9}$

A) $\frac{9}{2}$ would be $\cot A$ not sin A

B) $\frac{9}{2}$ would be $\cot A$ not tan A

C) $\frac{2}{9}$ would be $\tan A$ not cos A

D) correct

The answer is D.

Specific Strategies:

Which two sides are given in relation to angle A?

OPPOSITE & ADJACENT

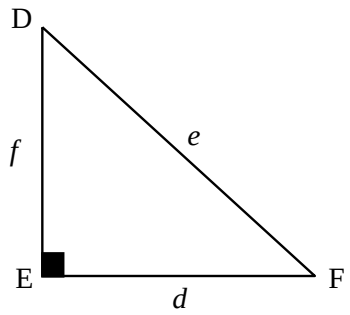
1. Identify the sides according to angle A.
2. Given the "knowns" (opposite and adjacent), select the correct trigonometric ratio.

Additional Resources:

Visions Volume 2, p. 84
Khan Academy Video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry
http://www.purplemath.com/modules/basirati.htm

QUESTION 94 5.1-MC-B

Answer and Solution:



A) cos D= $\frac{adjacent}{hypotenuse} = \frac{f}{e}$ incorrect

B) tan D= $\frac{opposite}{adjacent} = \frac{d}{f}$ incorrect

C) cos F= $\frac{adjacent}{hypotenuse} = \frac{d}{e}$ correct

D) sin F= $\frac{opposite}{hypotenuse} = \frac{f}{e}$ incorrect

The answer is C.

Specific Strategies:

1. Consider each ratio one at a time
2. Label sides according to angle D
3. Label sides according to angle F

Always solve for each choice (A, B, C and D).

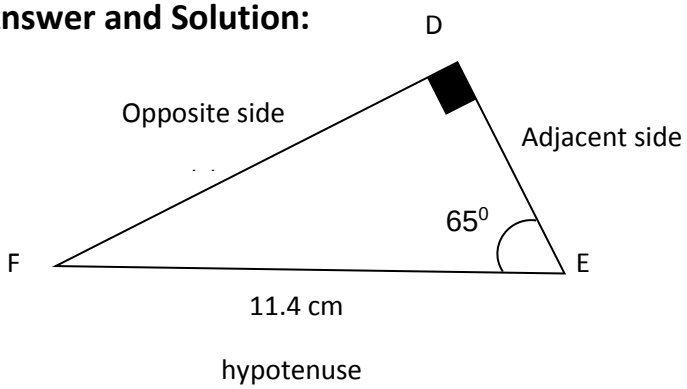
Don't stop when you think you have the correct answer.

Additional Resources:

Visions Volume 2, p. 84
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/basic-trigonometry
http://www.purplemath.com/modules/basirati.htm

QUESTION 95 5.1-MC-C

Answer and Solution:



- Length DF is the opposite side and the unknown (x)
- The hypotenuse is given
- Therefore, the correct trigonometric ratio to use is SIN (SOH)

MAKE SURE YOUR CALCULATOR IS IN DEGREES

$$\boldsymbol{sin\, E = \frac{opposite}{hypotenuse}}$$

$$\boldsymbol{sin\, 65 = \frac{x}{11.4}}$$

$$\boldsymbol{x = 11.4\, sin\, (65^o)}$$

$$\boldsymbol{x = 10.33}$$

A) This is the result of cos $65^o = \frac{x}{11.4}$ which is incorrect.

B) **10.33 cm is correct.**

C) This is the result of sin $65^o = \frac{11.4}{x}$ which is incorrect.

D) This is the result of tan $65^o = \frac{x}{11.4}$ which is incorrect.

The answer is B.

Specific Strategies:

Which angle is given?

Which side is given?

Which side is the unknown?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

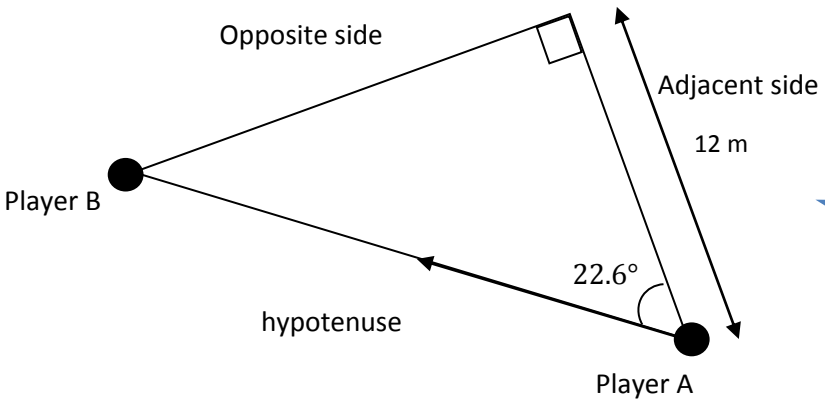
Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

Additional Resources:

Visions Volume 2, p. 84
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Sine, Cosine, and Tangent

Answer and Solution:



- Length of the adjacent side is given: 12 m
- The hypotenuse is the unknown (x)
- Therefore, the correct trigonometric ration to use is COS (CAH)

MAKE SURE YOUR CALCULATOR IS IN DEGREES

$$cos\ 22.6^0 = \frac{adjacent}{hypotenuse}$$

$$0.9232 = \frac{12}{x}$$

$$x = \frac{12}{0.9232} = 12.998\ or\ 13\ m$$

Answer: The distance traveled by the ball during the pass is 13.0 m.

Specific Strategies:

Which angle is given?

Which side is given?

Which side is the unknown?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

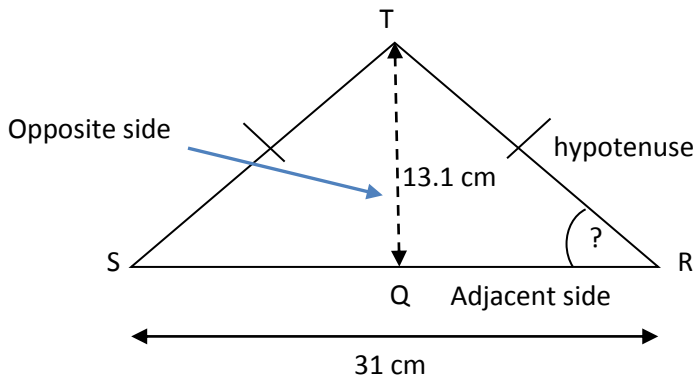
Remember: SOH CAH TOA

1. Label sides according to the given angle.
2. Select the appropriate trigonometric ratio (sin, cos or tan).
3. Solve for the unknown.

Additional Resources:

Visions Volume 2, p. 84; examples on p. 85
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle
http://www.purplemath.com/modules/basirati2.htm

Answer and Solution:



- The focus is on triangle TQR
- Triangle TQR is a right triangle because the height is perpendicular to the base.
- The measurement of QR is $31 \div 2$ because the triangle is isosceles
- Use trigonometric ratios
- The "knowns" are the opposite side and adjacent side to angle R
- The correct trigonometric ratio is Tan (TOA)

Remember to divide RS by two to obtain the value of the adjacent side of angle R. $31 \div 2 = 15.5$

$$\tan R = \frac{opposite}{adjacent}$$

$$\tan R = \frac{13.1}{15.5}$$

$$R = \tan^{-1} 0.8451$$

$$R = 40.2^0$$

The measure of angle R is 40.2^0.

Specific Strategies:

Which angle is given? or unknown?

Which sides are given?

Looking for an unknown in a right triangle means that you will need to use trigonometric ratios.

Remember: SOH CAH TOA

1. Label sides according to the given angle

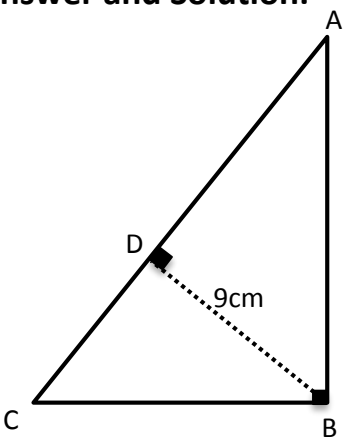
2. Select the appropriate trigonometric ratio (sin, cos or tan)

3. Solve for the unknown

Additional Resources:

Visions Volume 2, pp. 84-85
Khan Academy video: https://www.khanacademy.org/math/trigonometry/basic-trigonometry/basic_trig_ratios/v/example--trig-to-solve-the-sides-and-angles-of-a-right-triangle
http://www.purplemath.com/modules/basirati2.htm

Answer and Solution:



$\overline{AB}$ = 2 x $\overline{EB}$
= 2 x 8.82
= 17.64cm

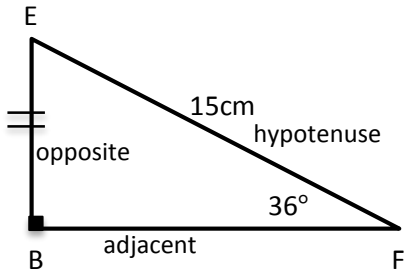
$\overline{BD}$ is an altitude of triangle ABC, so we can use the Pythagorean theorem to find $\overline{AD}$:
$\overline{AD}^2 + 9^2 = 17.64^2$
$\overline{AD} = 15.17$cm

We can also use metric relations:
$\overline{AB}^2 = \overline{AD}$ x $\overline{AC}$
$17.64^2 = 15.17$ x $\overline{AC}$
$\overline{AC}$ = 20.51 cm

$\overline{AC}$ x $\overline{DB}$ = $\overline{AB}$ x $\overline{BC}$
20.51 x 9 = 17.64 x $\overline{BC}$
$\overline{BC}$ = 10.46cm

Area = $\frac{base \times height}{2}$

$= \frac{10.46 \text{ x } 17.64}{2} = 91.32\ cm^2$



Start with triangle BEF:
Using the sin ratio, determine the length of side BE:

$\sin(36^0) = \frac{opposite}{hypotenuse} = \frac{\overline{BE}}{15}$

$15\times 0.5878 \approx 8.82cm$

Using the cos ratio (or Pythagorean Theorem), determine the length of side BF:

$\cos(36^0) = \frac{adjacent}{hypotenuse} = \frac{\overline{BF}}{15}$

$15\times 0.8090 \approx 12.14cm$

Determine the area of triangle BEF:

Area = $\frac{base \times height}{2}$

$= \frac{12.14 \text{ x } 8.82}{2} = 53.54\ cm^2$

Total area = 53.54cm^2 + 91.32cm^2 = **144.86 cm^2**

Specific Strategies:

Break the image down into two right triangles: ABC and BEF. Determine the area of each and then add them.

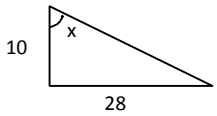
Area = $\frac{base \times height}{2}$

Formulas that may apply to right triangles:
- Pythagorean theorem
- Trigonometric ratios
- Metric relations

Additional Resources

Visions Volume____, Section 3.3, pp. 178-185
Visions Volume ____, Section 5.1, pp. 91-101

Answer and Solution:



$\tan x = \frac{28}{10}$

$x = \tan^{-1}(2.8) = 70.3°$

$? = 180° - 2(70.3°)$

$= 39.4°$

(You may instead get the result 39.3° which would be correct.)

A) Incorrect. Did $\tan x = \frac{10}{28}$ and forgot to subtract from 180°.

B) Correct answer.

C) Incorrect. Forgot to subtract from 180°.

D) Incorrect. Did $\tan x = \frac{10}{28}$ and subtracted from 180°.

The answer is B.

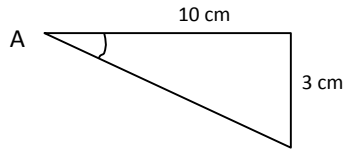
Suggested Strategies:

- Identify the triangles that you see – two right triangles and an isosceles triangle.
- Determine if the two right triangles are the same. (they are by SAS – the two legs and the right angles)
- Since you know the lengths of two sides of the right triangle, you can use a trig ratio to find the angle(s)
- You know that the sum of the angles along the side of the rectangle measuring 20 must add up to 180.
- Find *x* and subtract that twice from 180 and you'll get the measure of the missing angle.

Additional Resources:

Visions Volume 1 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:



$\tan A = \frac{3}{10}$

$\angle A = \tan^{-1}\left(\frac{3}{10}\right)$

$\angle A = 16.7°$

A) **Correct Answer.**

B) Incorrect. Used $\sin\frac{3}{10}$ instead of $\tan\frac{3}{10}$.

C) Incorrect. Used $\tan\frac{7}{20}$ instead of $\tan\frac{3}{10}$.

D) Incorrect. Used $\sin\frac{7}{20}$ instead of $\tan\frac{3}{10}$.

The answer is A.

Suggested Strategies:

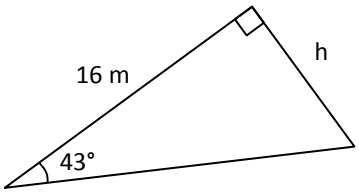
- Look at the illustration and find the right triangle(s) – you should see the two (implied) right triangles in the obtuse triangle of the flap.
- Since the 3 cm goes down the center, the top can be divided into 10 cm and 10 cm and you have two right triangles.
- (Notice that the 7 cm measure is extra information that you don't need.)
- Now you're ready to choose a trig ratio to solve for the missing angle. Tangent is what works in this case.
- Turn your paper around if the orientation of the triangle is confusing.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:

C) 119.4 m^2



$\tan 43° = \frac{h}{16}$

$0.19325 = \frac{h}{16}$

$h = 16 \times 0.9325$

$h = 14.92$ m

$A = \frac{16 \times 14.92}{2} = 119.4\ m^2$

A) Incorrect. $\sin 43^o$ was used instead of $\tan 43^o$.

B) Incorrect. $\cos 43^o$ was used instead of $\tan 43^o$.

C) Correct answer.

D) Incorrect. $\tan 47^o$ was used instead of $\tan 43^o$.

The answer is C.

Suggested Strategies:

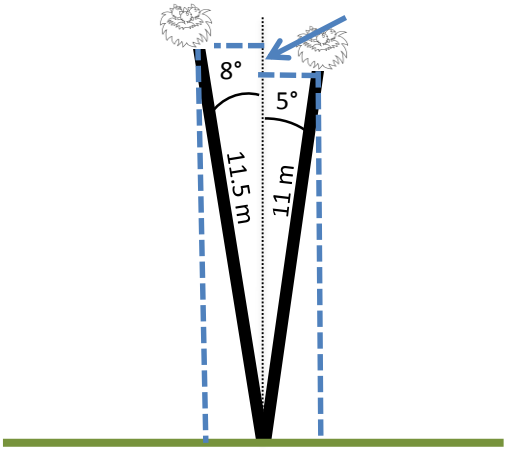
- Finding the area of a triangle means we need either the measure of all three sides (and use Hero's formula) or we need a base and its height.
- Here we have one side and an angle…in a right triangle so trig ratios can be used to find the measures of missing sides.
- If we consider the 16 m side as the base, the height is opposite the 43° angle.
- Tangent is the trig ratio to use in this case.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:

Difference in height between the bird nests = 0.43 m



Pole A	Pole B
$\cos 8° = \frac{Height_A}{11.5}$	$\cos 5° = \frac{Height_B}{11}$
$0.990 = \frac{Height_A}{11.5}$	$0.996(11) = Height_B$
$Height_A = 11.39$ m	$Height_B = 10.96$ m

Difference is 11.39 – 10.96 = <u>0.43 m</u>

The difference in height between the two bird nests is <u>0.43</u> m.

Suggested Strategies:

- Before you can find the difference in the heights of the bird nests, you need to find the height from the ground of each one.
- Height is a vertical distance so the height of each nest will be less than the length of the poles.
- You will have to identify right triangles and their elements before you can use trig ratios to solve for the missing measure.

Additional Resources:

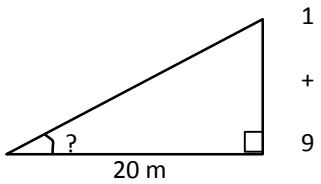
Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

QUESTION 103 5.2-SA-B

Answer and Solution:

$$sin30° = \frac{x}{18}$$
$$0.5 = \frac{x}{18}$$
x = 0.5 x 18 = 9 m

or the length of the side opposite a 30° angle in a 30°-60°-90° triangle is ½ the length of the hypotenuse



$$y = tan^{-1}\left(\frac{10}{20}\right)$$
$$y = tan^{-1}(0.5)$$
$$y = 26.6°$$

The angle of inclination of the left guy wire is 26.6°.

Suggested Strategies:

- Determine what you need to know about the left triangle in order to find the missing angle measure.
  - Either the hypotenuse (the length of the guy wire),
  - the other angle or
  - the height up the flagpole of the other guy wire.
- We only have enough information to find the last option.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

QUESTION 104 5.2-SA-C

Answer and Solution:

Use Sine Law to find $m\angle PQ$

$$\frac{p}{sinP} = \frac{q}{sinQ} = \frac{r}{sinR}$$

$$\frac{12\ cm}{sin42°} = \frac{15\ cm}{sinQ}$$

$$sinQ = \frac{15\ cm\ \cdot sin42°}{12\ cm}$$

$$sinQ = 0.8364132579$$

Find the inverse of Sine, use

$$sin^{-1}(0.8364132579) = 56.733°$$

Careful, this is not an obtuse angle!!

Extra step – since we are looking for an obtuse angle…

$$sinQ = sin(180° - Q)$$

$$sin(56.7633°) = sin(180° - 56.7633°)$$

$$180° - 56.7633° = 123.2367°$$

The measure of $\angle PQR$ is 123°.

Suggested Strategies:

- Since this is not a right triangle, you cannot use SOH CAH TOA
- Don't forget that when you use Sine Law to determine the measure of an obtuse angle, you must subtract your answer from 180.

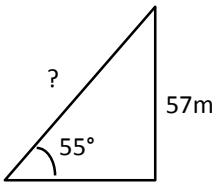
Additional Resources:

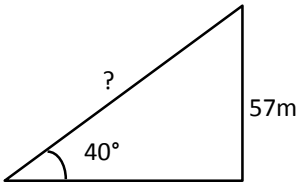
Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:

Height of kite where the string is attached is
60 m – 2 m = 58 m

Sally is holding the string 1 m up so height for calculation is 58 m – 1 m = 57 m





Length of string originally:

$$sin55° = \frac{57\ m}{l_1}$$
$$0.8191 = \frac{57\ m}{l_1}$$
$$l_1 = \frac{57\ m}{0.8191} = 69.58\ m$$

Length of string after the wind shift:

$$sin40° = \frac{57\ m}{l_2}$$
$$0.6428 = \frac{57\ m}{l_2}$$
$$l_2 = \frac{57}{0.6428} = 88.68\ m$$

88.68 m – 69.58 m = 19.1 m

Sally had to let out an additional 19.1 m of string to maintain the height of the kite.

Suggested Strategies:

- The tricky part of this question is trying to imagine what exactly is happening.
- From the diagram, you can probably see that triangles are involved; sketch them separately in a way that will make it easier to see how to set up the trig ratios
- Recognize that the string is the hypotenuse of these triangles and get the feel that the hypotenuse will be longer if the angle is smaller.
- Now figure out the two lengths and find the difference.

Additional Resources:

Visions Volume 2 p. 95 (Mathematical Knowledge Summary)
Explore Learning Gizmos, http://www.explorelearning.com/ look up: Proving Sine, Cosine and Tangent
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/ccgeometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:

$$\frac{a}{sinA} = \frac{c}{sinC}$$

$$\frac{12}{sinA} = \frac{15}{sin98}$$

$$12(sin98) = 15sinA$$

$$sinA = 0.79$$
$$A = 52.39^0$$

A) 53.1°; Did not use sine law, used $\sin CAB = \frac{a}{c}$ instead.

B) 52.4°; Correct Answer.

C) 38.7°; Did not use sine law, used $\tan CAB = \frac{a}{c}$ instead.

D) 36.8°; Did not use sine law, used $\cos CAB = \frac{a}{c}$ instead.

The answer is B.

Specific Strategies:

- Notice that this is <u>not</u> a right angle (90^0) triangle.
- You <u>cannot</u> apply SOH, CAH, TOA.
- You should use Sine law.
- Color code, highlight or match the angles with their corresponding sides.
- Apply Sine law formula.

Additional Resources:

Visions Volume 2, Section 5.3, p. 108
Khan Academy video: https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines Law of Sines

QUESTION 107 5.3-MC-B

Answer and Solution:

$$p = \frac{(a+b+c)}{2}$$

$$p = \frac{(21+18+15)}{2} = 27$$

$$A = \sqrt{p(p-a)(p-b)(p-c)}$$
$$A = \sqrt{27(27-21)(27-18)(27-15)}$$
$$A = \sqrt{27(6)(9)(12)}$$
$$A = \sqrt{17\,496}$$
$$A = 132.3\ m^2$$

Area = 132.3 m^2

A) 7.35 m^2 ; added the values under the radical

B) 25.5 m^2; Forgot to multiply everything by p under the radical.

C) 132.3 m^2; Correct Answer

D) 187.1 m^2; Multiplied everything by the perimeter instead of the half perimeter

The answer is C.

Specific Strategies:

Identify the values of *a*, *b* and *c* according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for *a*, *b*, *c*, and *p* in Hero's formula.

Additional Resources:

Visions Volume 2, Section 5.3, p. 108
Khan Academy video: https://www.khanacademy.org/math/geometry/basic-geometry/heron_formulatutorial/v/ heron-s-formula Heron's Formula

QUESTION 108 5.3-SA-A

Answer and Solution:

Knowing that angle C is 55°, that side C has a length of 10 and side b a length of 7, we can calculate the size of angle B using sine law.

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

To solve the problem, we need three of the four values in any given equality. Since we know b, c and sin C, then:

$$\frac{b}{sinB} = \frac{c}{sinC}$$

Rearranging the equation we get:

$$\sin B = \frac{bsinC}{c}$$

Substituting the corresponding values, we get:

$$\sin B = \frac{7sin55^o}{10}$$

$$\sin B = 0.573406431$$

Finding the inverse of sin, $\sin^{-1}$, we can get the measure of the angle:

$$B = \sin^{-1} 0.573406431$$

$$B = 35^o$$

Answer: Angle B = 35^0

Specific Strategies:

- Notice that this is <u>not</u> a right angle (90^0) triangle
- You <u>cannot</u> apply SOH, CAH, TOA
- You should use Sine law
- Color code, highlight or match the angles with their corresponding sides
- Apply Sine law formula

Additional Resources:

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)
Khan Academy video: https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines Law of Sines

Answer and Solution:

Before starting with Hero's formula, we must first determine the half-perimeter p of the triangle.

$$Since\ p = \frac{(a+b+c)}{2}$$
$$where\ a = 7\ cm, b = 5\ cm\ and\ c = 6\ cm$$
$$we\ know\ that\ p = \frac{(7\ cm + 5\ cm + 6\ cm)}{2} = 9\ cm$$
$$Now\ using\ the\ formula\ we\ get:$$

$$A = \sqrt{p(p-a)(p-b)(p-c)}$$
$$A = \sqrt{9\ (9-7)(9-5)(9-6)}$$
$$A = \sqrt{9(2)(4)(3)}$$
$$A = \sqrt{216}$$
$$A = 14.7\ cm^2$$

Area = 14.7 cm^2

The answer is 14.7 cm^2.

Specific Strategies:

Identify the values of *a*, *b* and *c* according to the diagram provided.

Calculate the value of the half perimeter.

Substitute the values for *a*, *b*, *c*, and *p* in Hero's formula.

Additional Resources:

Visions Volume 2 Activity 2 p. 105 and Mathematical Knowledge p. 108
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry Sine, Cosine and Tangent Trigonometric Functions

Answer and Solution:

Knowing that angle B is 53°, that side b has a length of 6 and angle C is 70°, we can calculate the length of side c using sine law.

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

To solve the problem, we need three of the four values in any given equality. Since we know b, sin B and sin C, then:

$$\frac{b}{sinB} = \frac{c}{sinC}$$

Rearranging the equation we get:

$$c = \frac{bsinC}{sinB}$$

Substituting the corresponding values, we get:

$$c = \frac{6sin70^o}{sin53^o}$$

$$c = 7.06$$

Since side c corresponds to AB, the length of segment AB is 7.06.

Answer: the length of segment AB is 7.06.

Specific Strategies:

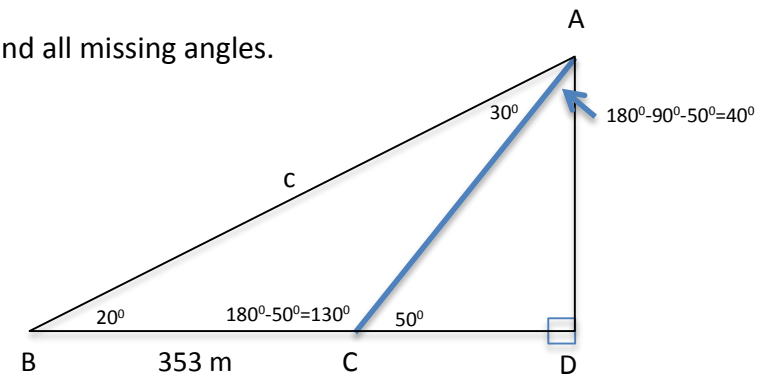
- Notice that this is <u>not</u> a right angle (90^0) triangle
- You <u>cannot</u> apply SOH, CAH, TOA
- You should use Sine law
- Color code, highlight or match the angles with their corresponding sides
- Apply Sine law formula

Additional Resources:

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)
Khan Academy video: http://www.khanacademy.org/math/geometry/right_triangles_topic/cc-geometry-trig/v/basic-trigonometry Sine, cosine and tangent trigonometric functions

Answer and Solution:

Find all missing angles.



Apply Sine law to find c.

$$\frac{a}{sinA} = \frac{c}{sinC}$$

$$\frac{353}{sin30} = \frac{c}{sin130}$$

$$c = 541\ m$$

Apply trig to find $\overline{AD}$ (height of Space Needle).

$$sin20 = \frac{\overline{AD}}{541}$$

$$\overline{AD} = 185\ m$$

✓ Yes, his estimation is correct.
☐ No, his estimation is not correct.

Reason: The clinometer measures yield a height of 185 m for the Space Needle. This falls within Phil's estimation of 182 m to 188 m.

Specific Strategies:

1. Find all the missing angles you can
2. Notice triangle ABD is a right angle triangle composed of two other triangles
3. Triangle ABC is not a right angle triangle. So, apply Sine law
4. Triangle ACD is a right angle triangle. So, apply SOH, CAH TOA

Additional Resources:

Visions Volume 2, Section 5.3, p. 103 (Activity 1: Sine Law)
Khan Academy video: https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/law-sines-cosines/v/law-of-sines Law of Sines