I. Model Problems.
II. Calculate Side using Law of Cosines
III. Calculate Angle using Law of Cosines
IV. Mixed (angle and side)problems
V. Challenge questions
VI. Answer Key

Web Resources

Video explanation of Law of Cosines with additional practice problems
www.mathwarehouse.com/trigonometry/law-of-cosines-formula-examples.php
Pictures Law of Cosines www.mathwarehouse.com/trigonometry/images/law-of-cosines/

Law of Cosines
For any ΔABC:
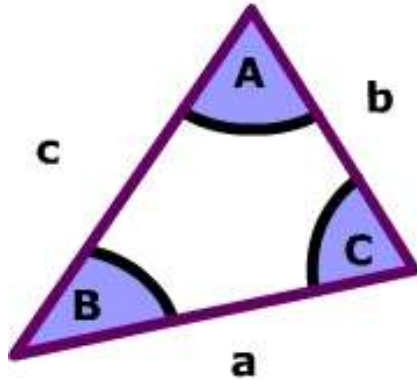
Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(A)$$
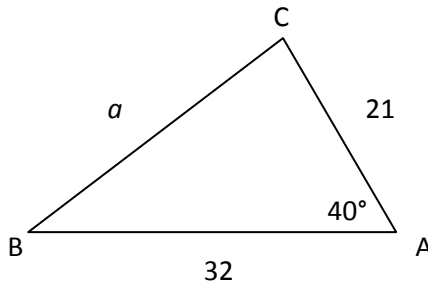$$b^2 = a^2 + c^2 - 2ac \cdot \cos(B)$$
$$c^2 = b^2 + b^2 - 2bc \cdot \cos(c)$$

I. Model Problems

In the following example you will find the length of a side of a triangle using Law of Cosines.

Example 1:
Find the length of a.

Write down known.	$b = 21, c = 32,\ \mathrm{m}\angle A = 40°$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$
Substitute.	$a^2 = (21)^2 + (32)^2 - 2(21)(32) \cos 40°$
Simplify.	$a^2 = 441 + 1024 - 1344 \cos 40°$
	$\sqrt{a^2} = \sqrt{441 + 1024 - 1344 \cos 40°}$
Round to the nearest hundredth.	$a \approx 20.87$

In the following example you will find the measure of an angle of a triangle using Law of Cosines.

Example 2:
Find m∠A.
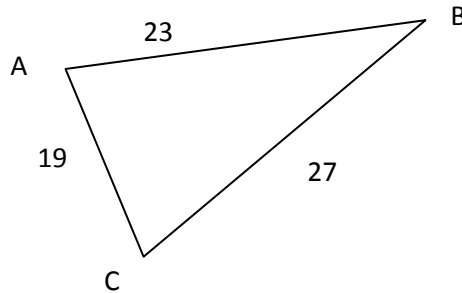
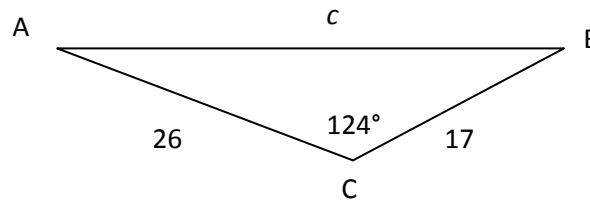| Write down known. | $a = 27, b = 19, c = 23$ |
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$
Substitute.	$(27)^2 = (19)^2 + (23)^2 - 2(19)(23) \cos A$
Simplify.	$729 = 361 + 529 - 874 \cos A$
	$729 = 1465 - 874 \cos A$
Isolate $\cos A$.	$-736 = -874 \cos A$
Find the inverse.	$\dfrac{736}{874} = \cos A$
Round to the nearest hundredth.	$m\angle A \approx 79.38°$

In the following example you will find the length of a side of a triangle using Law of Cosines.

Example 3:
For △ABC, find the length of c given $a = 17, b = 26$, and m∠C = 124°.
Draw and label a triangle.

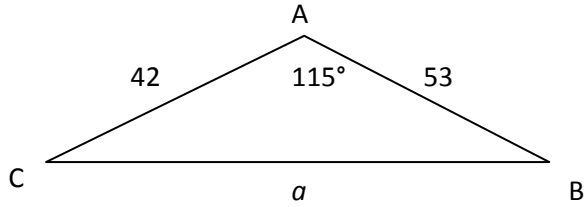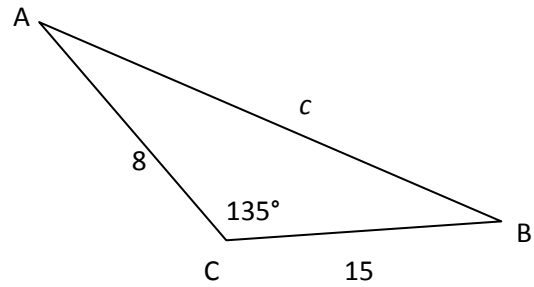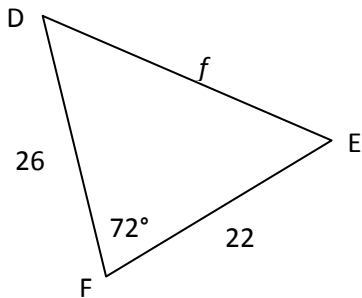| Write down known. | $a = 17, b = 26, m\angle C = 40°$ |
Alternative form of Law of Cosines	$c^2 = a^2 + b^2 - 2ab \cos c$
Substitute.	$c^2 = (17)^2 + (26)^2 - 2(17)(26) \cos 124°$
Simplify.	$c^2 = 289 + 676 - 884 \cos 124°$
	$\sqrt{c^2} = \sqrt{289 + 676 - 884 \cos 124°}$
Round to the nearest hundredth.	$c \approx 38.20$

II. Find the length of a side using Law of Cosines.

1. For ΔABC find a to the nearest hundredth.

2. For ΔABC find c to the nearest hundredth.

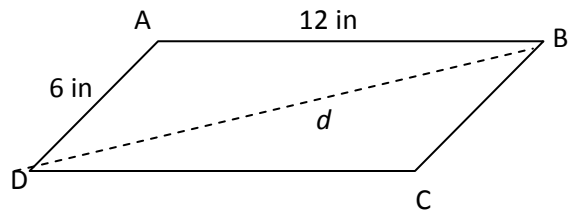3. For ΔDEF find f to the nearest hundredth.

4. For ΔABC find the length of a to the nearest hundredth, given $b = 8, c = 23$, and m∠A = 29°.

5. For ΔABC find the length of c to the nearest hundredth, given $a = 54, b = 47$, and m∠C = 85°.

6. Find the length of the diagonal, d, of the parallelogram below to the nearest inch.

7. A regular hexagon has side lengths of 15 centimeters and angles that measure 120°. Find FB to the nearest centimeter.
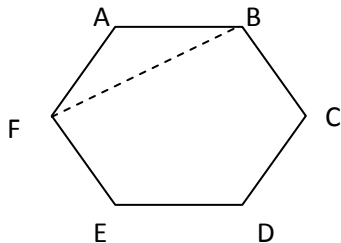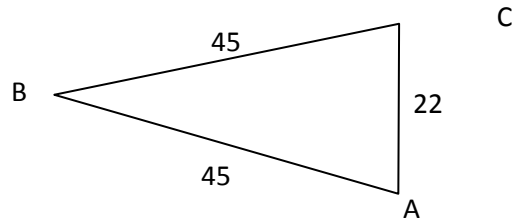
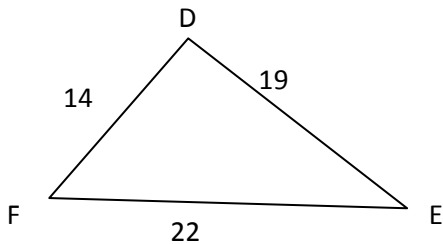**III. Find the measure of an angle using Law of Cosines.**

8. For ΔABC find m∠A to the nearest tenth of a degree.

19
A ————————— B
17
23
C

9. For ΔABC find m∠B to the nearest tenth of a degree.

C
45
B
22
45
A
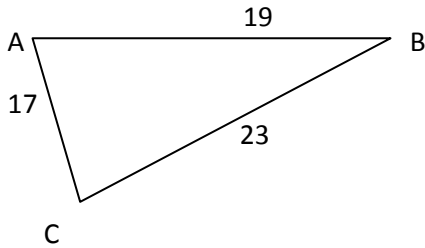
10. For ΔDEF find m∠E to the nearest tenth of a degree.

D
14
19
F
22
E

11. For ΔABC find m∠B to the nearest tenth, given $a = 7, b = 6,$ and $c = 5$.

12. For ΔDEF find m∠F to the nearest tenth, given $d = 38, e = 42,$ and $f = 47$.

13. Find m∠P for the parallelogram below to the tenth of a degree.
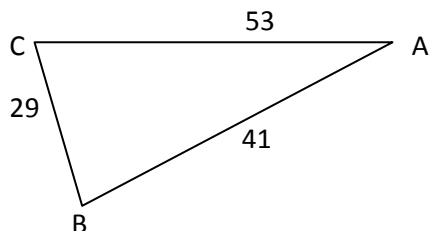
S
26 in
P
28 in
37 in
R
Q
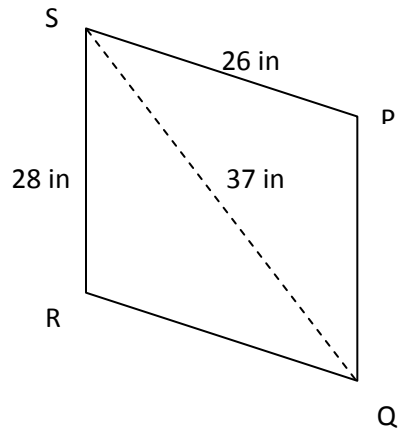
14. A rhombus has side lengths of 25 inches. The diagonal opposite the obtuse angles is 45 inches. What is the measure of the obtuse angle to the nearest degree?
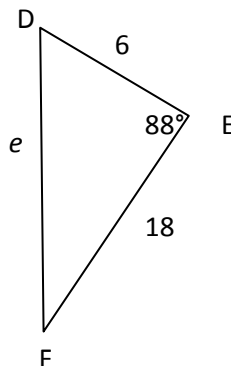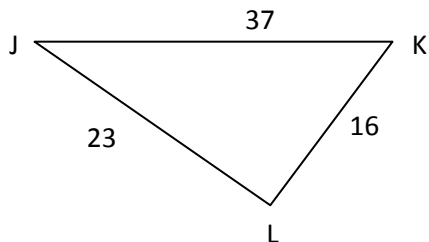
45 in
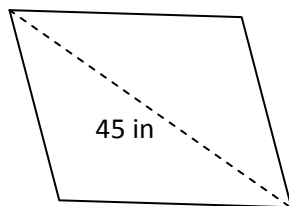
IV. Using Law of Cosines.

15. For ΔABC find m∠B to the nearest tenth of a degree.

53
C ——————— A
29
41
B

16. For ΔDEF find e to the nearest hundredth.

D
6
88° E
e
18
F

17. For ΔJKL find m∠K to the nearest tenth of a degree.

37
J ——————— K
23
16
L

18. For ΔXYZ find the length of z to the nearest hundredth, given $x = 81, y = 75$, and m∠Z = 42°.

19. For ΔDEF find the length of e to the nearest hundredth, given $d = 34, f = 42$, and m∠E = 24°.

20. For ΔABC find the length of b to the nearest hundredth, given $a = 27, c = 20$, and m∠B = 74°.

21. For ΔJKL find m∠K to the nearest tenth, given $j = 16$, $k = 19$, and $l = 27$.

V. Challenge Problems

22. Peter has three sticks measuring 19 inches, 23 inches, and 27 inches. He lays them down to form a triangle. Find the measure of the angle enclosed by the 19 inch and 23 inch sides to the nearest degree.

23. Mary is orienteering across a large flat plain from Marker A to Marker B which are 4 miles apart. After walking 1.8 miles she realizes she is 6° off-course. To the nearest tenth of a mile, how far from Marker B is she when she realizes her error?
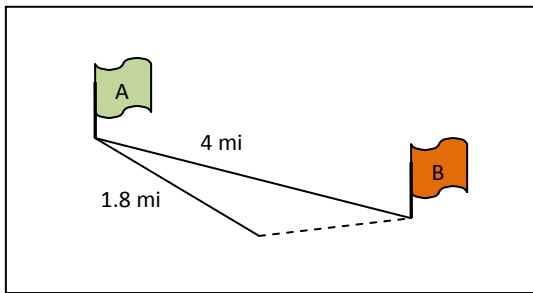
24. A navigator plots the course a plane is currently traveling. The plane is 300 miles from its destination. If it continues on its current course it will travel 325 miles and end up 125 miles due south of its destination. To the nearest degree, how many degrees is the plane off course?

25. For ΔABC find m∠B to the nearest degree.
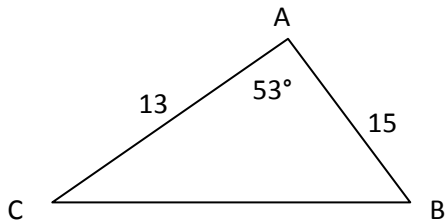
VI. Answer Key

1. $a \approx 80.34$
2. $c \approx 21.42$
3. $f \approx 28.40$
4. $a \approx 16.47$
5. $c \approx 68.43$
6. $d \approx 17$ in
7. FB ≈ 26 cm
8. m∠A $\approx 79.2°$
9. m∠B $\approx 28.3°$
10. m∠E $\approx 39.1°$
11. m∠B $\approx 57.1°$
12. m∠F $\approx 71.8°$
13. m∠P $\approx 86.4°$
14. $128°$
15. m∠B $\approx 96.9°$
16. $e \approx 18.77$
17. m∠K $\approx 22.2°$
18. $z \approx 56.19$
19. $e \approx 17.63$
20. $b \approx 28.83$
21. m∠K $\approx 43.76°$
22. $79°$
23. 2.2 miles
24. $23°$
25. m∠B $\approx 55°$