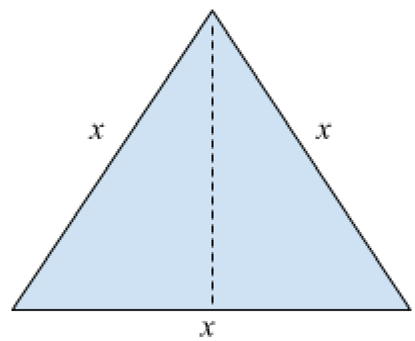
Introduction to Trigonometry

Trigonometry is the study of measurements in triangles… that's it. Done!

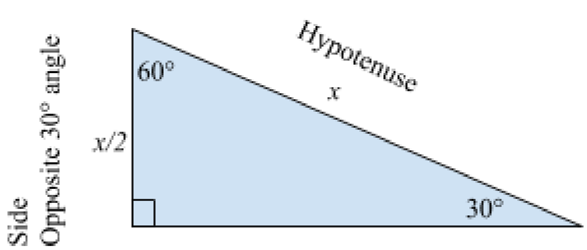
Okay, maybe not.

It is based on SIMILAR RIGHT TRIANGLES and the fact that any two right triangles with one congruent acute angle are similar, therefore the ratio of their sides is the same.

Let's begin with the equilateral triangle shown below, we'll say that it's side length is x. (and we know each angle is 60°)



We're going to cut this in half and one of these triangles will look like this:



You'll notice that the side opposite the 30° angle will ALWAYS measure half of the hypotenuse.

Our first trigonometric ratio is called the SINE RATIO (abbreviated "sin"), it is the ratio of the measure of the side opposite the **reference angle** (in the above case 30°) divided by the measure of the hypotenuse.

Using the diagram above you may realize that $\sin 30° = \dfrac{x/2}{x}$ or $\dfrac{0.5x}{x}$

Reduced, we get $\sin 30° = 0.5$

You can verify this on your calculator: look for the “sin” button and make sure your calculator is in “degree” mode (you should see a little “d” or “deg” somewhere on your calculator’s display). Enter sin 30 = and you should get 0.5 (or 30 sin depending on the type of calculator you have) If you don’t then you are not in degree mode… reset your calculator!

This is one of the few trig ratios that works out nicely. We usually just keep the values on the calculator & keep working with them, but if you feel the need to write down the ratios you MUST KEEP 4 decimal places!

The other two trig ratios are:

The **COSINE RATIO** (abbreviated “cos) is the ratio of the measure of the side adjacent to the **reference angle** (this is the side which is touching the reference angle, but is not the hypotenuse) divided by the measure of the hypotenuse.

The **TANGENT RATIO** (abbreviate “tan”) is the ratio ratio of the measure of the side opposite the **reference angle** divided by ratio of the measure of the side adjacent to the reference angle.

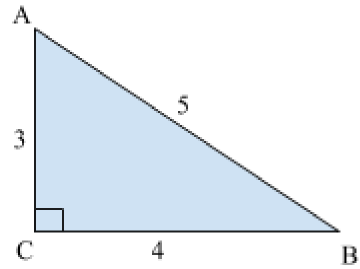
NOTE that the hypotenuse is the longest side of the right triangle and it is always found opposite the 90° angle, that leaves two other sides, the **opposite** and **adjacent** sides. The labels for these two sides **depend on the reference angle**. The reference angle CANNOT be the 90° angle.

The mnemonic $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$ is used by many to remember the ratios.

You’ll notice that the sine, cosine and tangent ratios require a **REFERENCE ANGLE**, without a reference angle you cannot label the sides and you would not know if you were dealing with similar triangles.

For instance, if we were to write sin 10°, then we are referring to a right triangle with angles 10°, 80° and 90°, specifically to the ratio of the side opposite the 10° angle divided by the hypotenuse.

Example: Given the following triangle determine the ratios sin A, cos A, tan A, sin B, cos B and tan B



$$\sin A = \frac{\text{measure of side opposite to } \angle A}{\text{hypotenuse}}$$

$$\sin A = \frac{4}{5}$$

$$\cos A = \frac{\text{measure of side adjacent to } \angle A}{\text{hypotenuse}}$$

$$\cos A = \frac{3}{5}$$

$$\tan A = \frac{\text{measure of side opposite to } \angle A}{\text{measure of side adjacent to } \angle A}$$

$$\tan A = \frac{4}{3}$$

$$\sin B = \frac{\text{measure of side opposite to } \angle B}{\text{hypotenuse}}$$

$$\sin B = \frac{3}{5}$$

$$\cos B = \frac{\text{measure of side adjacent to } \angle B}{\text{hypotenuse}}$$

$$\cos B = \frac{4}{5}$$

$$\tan B = \frac{\text{measure of side opposite to } \angle B}{\text{measure of side adjacent to } \angle B}$$

$$\tan B = \frac{3}{4}$$

Notice that sin A = cos B and that sin B = cos A, this is because the side opposite $\angle A$ is the side adjacent to $\angle B$ and vice versa.