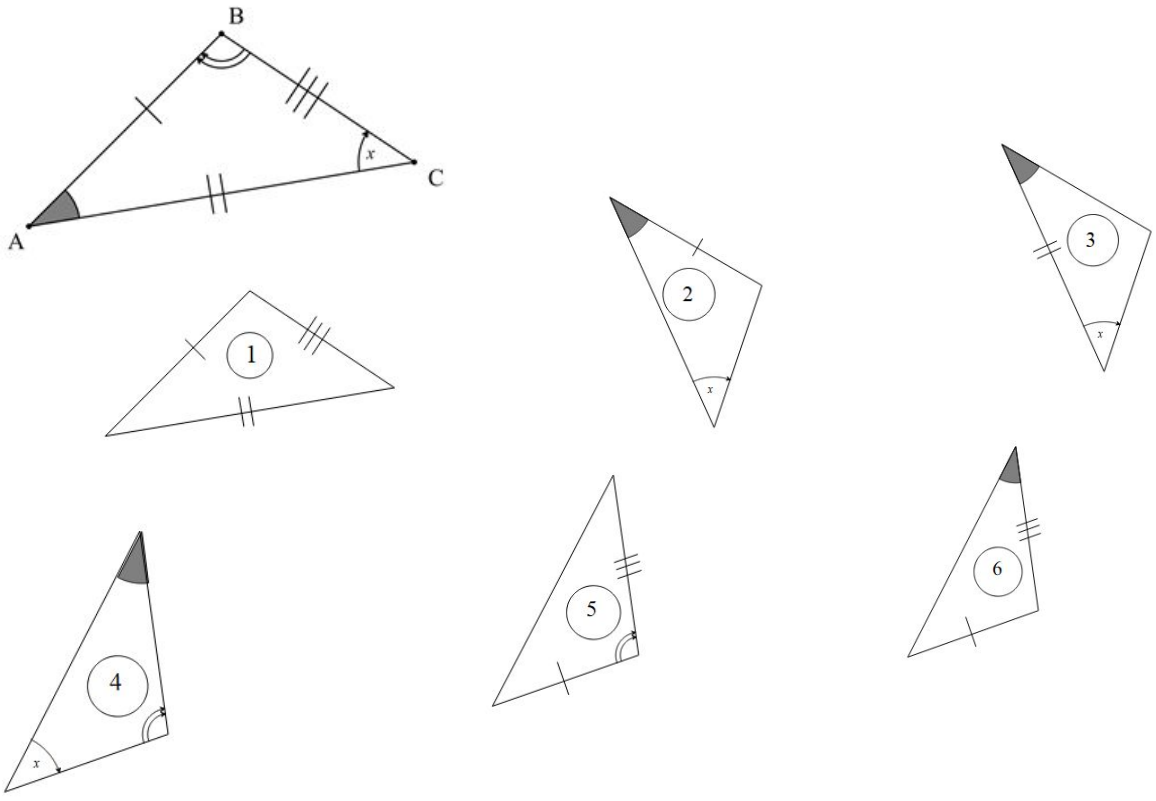
Isometric Triangles (aka: Congruent triangles) - exact same shape & size

- All corresponding sides are congruent
- All corresponding angles are congruent

Minimum Conditions for Concluding that Two Triangles are Isometric

1) Side-Side-Side (SSS)
   If all three pairs of corresponding sides are congruent, then the triangles are isometric.
2) Side-Angle-Side (SAS)
   If two pair of corresponding sides are congruent AND the **contained** corresponding angles are congruent (angles between the two sides), then the triangles are isometric.
3) Angle-Side-Angle (ASA)
   If two pair of corresponding angles are congruent AND the contained corresponding sides are congruent (sides between the two angles), then the triangles are isometric.

Example which of the following triangles must be isometric to $\triangle ABC$ and why? (**note**: do not go by looks, you must go strictly by the markings on the triangles!)



Answers: #1) yes, SSS #2) no the side must be between the two angles #3) yes, ASA
#4) no need a side #5) yes, SAS #6) no, angle must be between the two sides

Similar Triangles - exact same shape, different size

- All corresponding sides are **PROPORTIONAL**
- All corresponding angles are congruent

Minimum Conditions for Concluding that Two Triangles are Similar

4) Side-Side-Side (SSS)
   If all three pairs of corresponding sides are **proportional**, then the triangles are similar.
5) Side-Angle-Side (SAS)
   If two pair of corresponding sides are **proportional** AND the **contained** corresponding angles are **congruent**, then the triangles are similar.
6) Angle- Angle (AA)
   If two pair of corresponding angles are **congruent** then the triangles are similar.

***NOTE: When looking for proportional sides you must compare smallest side to smallest side, longest side to longest side and for 3 pairs of sides, medium side to medium side.

SIMILARITY STATEMENTS (the symbol for similarity is $\sim$)

A similarity statement indicates exactly which angles are congruent to each other (these are called "corresponding angles" and it therefore indicates which sides correspond to each other to form a proportion.

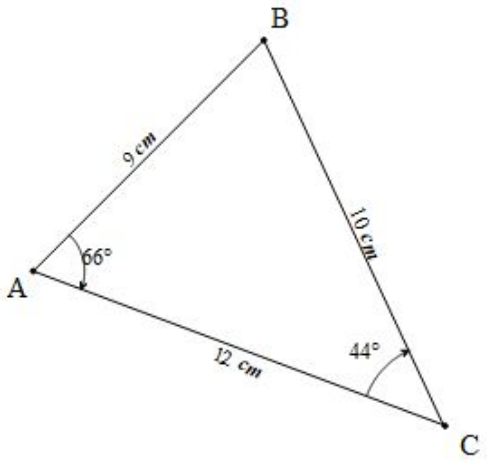
Ex. If $\Delta ABC \sim \Delta DMP$ then $\angle A$ *corresponds to* $\angle D$ (first letter with first letter,
$\angle B$ *corresponds to* $\angle M$
$\angle C$ *corresponds to* $\angle P$
(first letter with first letter, second with second and third with third)

Also: Side AB corresponds to side DM
Side BC corresponds to side MP
Side AC corresponds to side DP

This means that the proportion for the sides would be:

$$\frac{\overline{AB}}{\overline{DM}} = \frac{\overline{BC}}{\overline{MP}} = \frac{\overline{AC}}{\overline{DP}}$$

Example:

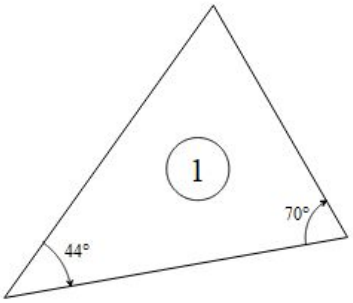


Determine which of the following triangles must be similar to $\triangle ABC$ and explain why or why not?

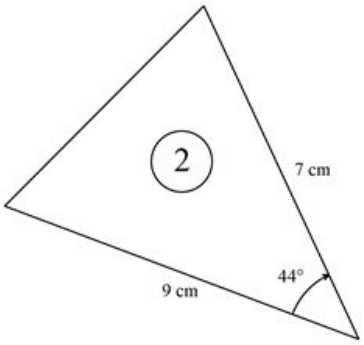
Note: if you have 2 angles you should always calculate the missing angle

$m\angle ABC = 180° - (66° + 44°)$

$m\angle ABC = 70°$



Since the original triangle has a $44°$ angle and a $70°$ then triangle 1 is similar by AA



Since this triangle and the original triangle each have a $44°$ angle we must check to See if the sides next to the $44°$ angle are proportional.

$\frac{12\ cm}{9\ cm} = \frac{10\ cm}{7\ cm}$?

$1.\overline{3} \neq 1.\overline{428571}$ they are not proportional, so the triangles are **not** similar

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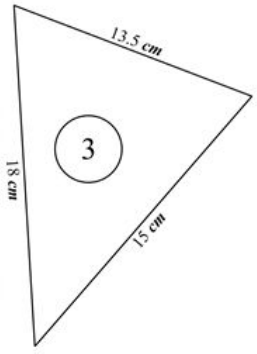
Given 3 sides we must see if they are proportional:

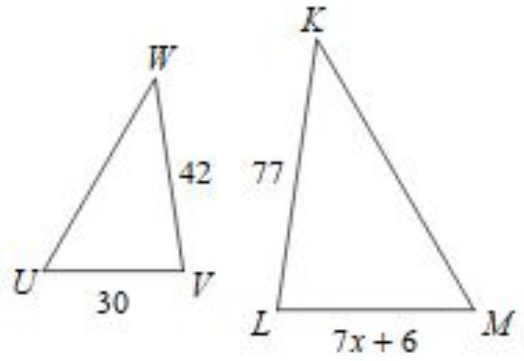


$\frac{12\ cm}{18\ cm} = \frac{10\ cm}{15\ cm} = \frac{9\ cm}{13.5\ cm}$?

$0.\overline{6} = 0.\overline{6} = 0.\overline{6}$ they are proportional, so the triangles are similar because of SSS.

Example: Solve for x.

$\triangle KLM \sim \triangle WVU$



Based on the **similarity statement** we can write the proportion:

$\frac{\overline{KL}}{\overline{WV}} = \frac{\overline{LM}}{\overline{VU}} = \frac{\overline{KM}}{\overline{WU}}$ substituting for the sides that are given, we get

$\frac{77}{42} = \frac{7x+6}{30}$ and then cross-multiplying we get

$42\,(7x + 6) = 77\,(30)$ distributing and multiplying…

$294x + 252 = 2310$ subtract 252 from both sides…

$294x = 2058$ dividing both sides by 294...

$x = 7$