Math 4 CST – Midyear Review (December)

1 Two lawn maintenance workers offer their services for the summer to the residents in a certain part of town. The first charges $1 per 40 square metres of lawn. The second charges 2.5 cents per square metre of lawn.

For a lawn that is 400 square metres, which offer is the best deal?

Show all the work needed to solve the problem.

2 A restaurant owner must decide which of three radio stations will be given his publicity campaign. Each of these stations has the same number of listeners. The restaurant owner has $5000 to spend on publicity.

- The first station charges $1000 for the advertisement plus $100 for every 30 seconds on the air.
- The second station charges $2000 for the advertisement plus $50 for every 30 seconds on the air.
- The third station doesn't charge for the advertisement but charges $150 for every 30 seconds on the air.

Which radio station should the restaurant owner choose if he wants the most air time for his money?

Show all the work needed to solve the problem.

3 Two stores decide to liquidate a product that they normally sell for the same price. The first advertises that, from the first day of the liquidation sale, it will take 10% off the initial price each day until a discount of 50 % is reached. At the second store, each day 10 % will be taken off the price announced the previous day until a discount of 50% is reached. The liquidation sales start on the same day.

What is the difference in per cent between the sale prices of the product in the two stores on the 4th day of the liquidation sale?

Show all the work needed to solve the problem.

4 | Annie, Gaby and Eric love to play pool. They play in various pool halls in town.

The cost of playing at Annie's favourite pool hall is represented by the adjacent graph.
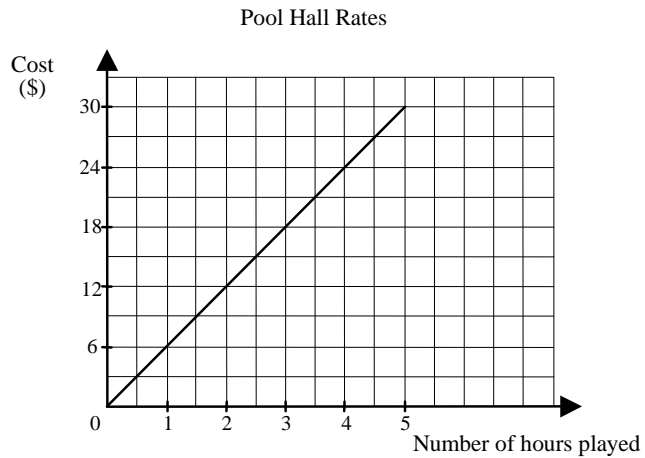
Pool Hall Rates

The cost of playing at Gaby's favourite pool hall is shown on the table of values below:

Pool Hall Rates

Number of hours played	0	1	2	3	4	5
Cost $	15.00	15.00	15.00	15.00	15.00	15.00

At Eric's favourite place, they charge a flat rate of $6.00 per table per hour.

If a game lasts less that two and a half hours, whose favourite pool hall offers the best deal?

Justify your answer.

5 | A lab technician notes that the number of type A bacteria doubles every hour whereas the number of type B bacteria triples every hour.
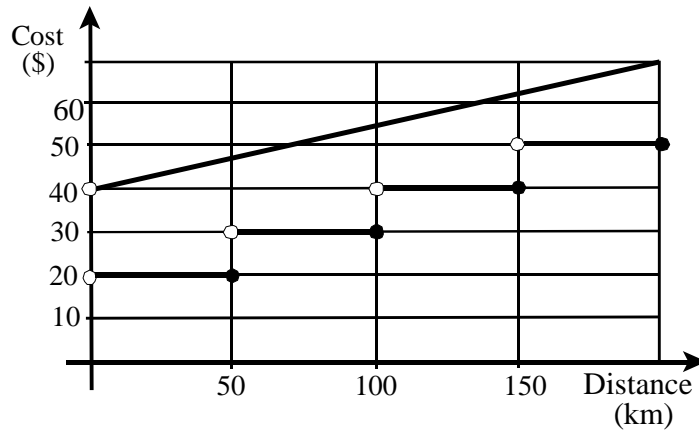
At the outset there are 1000 of type A bacteria and 500 of type B bacteria.

Which of the two bacteria will be more numerous after five hours?

Show all your work.

6 A car leasing company has two rental plans.

The following graph shows the relationship between the distance travelled and the cost of renting a car under each plan.

A customer drives 240 km in a rented car.

What is the exact difference in cost between the two plans?

Show all your work.

7 The following brainteaser came up during a t.v. quiz show.

 M_1:   An amount of $0.01 triples every day.

   M_2:   An amount of $1.00 doubles every day.

After 5 days, what will be the difference between the two amounts?

Show all your work.

8   Paul has the option of renting a car for one day from one of three car rental agencies. The amount they charge is illustrated in three different ways.

Company A uses the following graph to display its rates:

Rental Cost
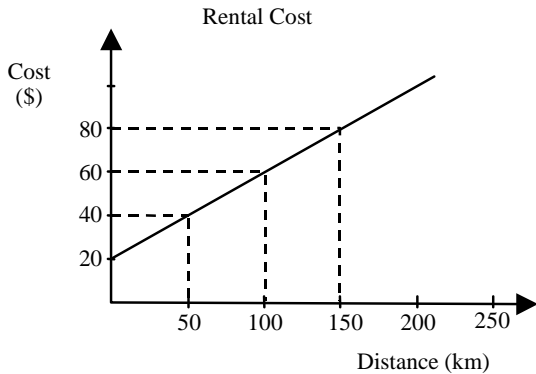
Company B has the following sign posted:

> We charge a flat rate of $115 a day regardless of distance travelled.

Company C uses the following table to display its costs:

Distance (km)	]0, 50[	[50, 100[	[100, 150[	[150, 200[	...
Cost ($)	80	85	90	95	...

What would be the difference in the cost between leasing a car from the most expensive company and the least expensive company if Paul drives a distance of 280 km?

Show all your work.

9   The value of a $60 000 car diminishes at a rate of 20% a year. However, the value of a $40 000 truck diminishes at a rate of 10% a year.

The two vehicles are going to be sold after 5 years.

At resale time, which vehicle will be worth the most?

Show all your work.

10 | Nancy invested $100 in company A and $75 in company B. After a year, she checked to see how her investments were doing.

- The value of her shares in company A increased at a rate of 5% a month.
- The value of her shares in company B decreased according to the pattern below:

Month	0	3	6	9	12
Amount ($)	75	70.59	66.44	62.53	58.85

After twelve months, what was the total profit that Nancy earned from her two investments?
Show all your work.

11 | Mr. Chen has just had a swimming pool installed. The pool can hold 32 000 litres of water.

The installers start filling the pool at 10:00 a.m. using a pump capable of discharging water at the rate of 2000 L an hour.

At 11:00 a.m., they add a second pump that discharges 3000 L of water an hour.

At what time will Mr. Chen's pool be filled to capacity?
Show all your work.

12 | In two video rental stores, customers must pay a fine if they return a movie after the due date.

To determine the total amount charged for renting a movie when it is returned after the due date, store A uses the following rule:

$$y_A = 4(1.15)^x$$

Where y_A represents the total amount charged, in dollars, for renting a movie and x represents the number of days after the due date.

Store B charges a flat rate for renting a movie plus a fine for each day after the due date. The following table of values gives examples of the total amounts charged by store B.

Number of days after the due date	Total amount charged
1	$4.75
5	$7.75
10	$11.50

What is the difference between the total amounts charged by these stores for a movie returned 7 days after the due date?

Show all your work.

13 Peter wants to rent a car. He checks the rental costs at two car rental companies. The graph below shows the relation between the rental cost of each company and the distance travelled. Peter plans to drive 120 km.

The Cost of Renting a Car
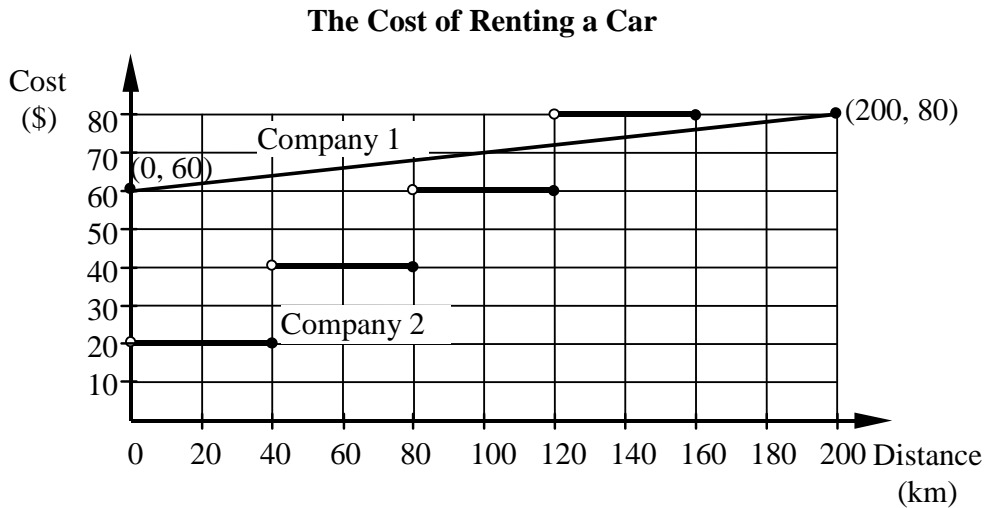
What is the difference in the rental cost between the two companies?
Show all your work.

14 The Varlok Ski Resort rents snowboards.

Rental prices vary according to the number of hours the snowboard is rented.

- two hours or less: $8
- More than 2 hours but less than or equal to 4 hours: $14
- More than 4 hours but less than or equal to 6 hours: $18
- More than 6 hours but less than or equal to 9 hours: $20

Construct a graph that represents this situation. (Be sure to label the axes.)
Show all your work.

15 In 2005, a doctor compared the number of people infected with AIDS in two African countries. In Country A, 1100 people had AIDS. This number is expected to increase by 6% per year. In Country B, 800 people were infected with AIDS. This number is expected to increase by 11% per year.
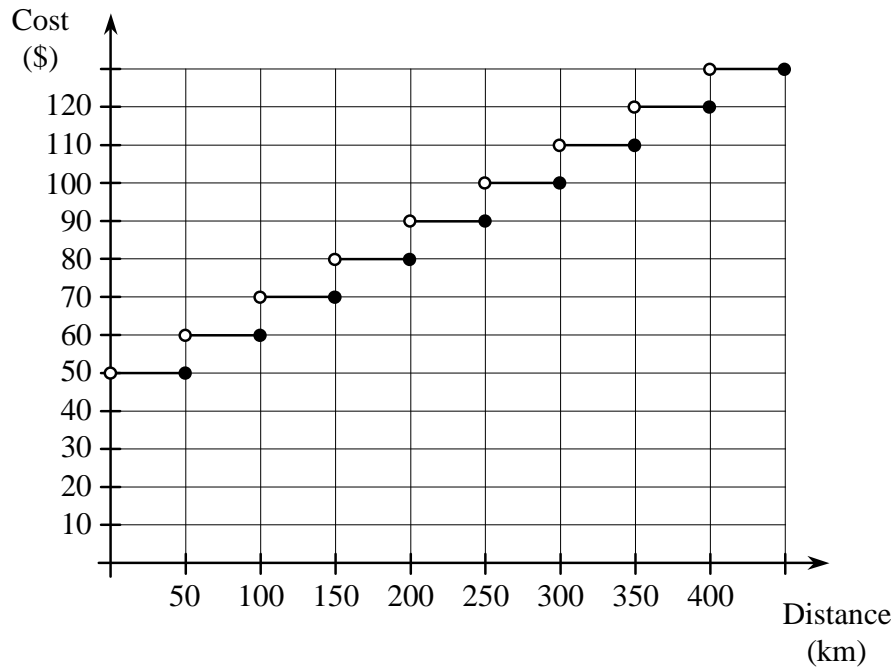
In 5 years, which country will have more people infected with AIDS?

Show all your work.

16 George wants to rent a car for a trip. The car rental company asks him to choose one of the following two rate plans.

Plan A

Under Plan A, the rental cost based on distance travelled is represented by the following graph.

Plan B

Under Plan B, the rental cost is determined as follows:

• $0.50/km for the first 120 kilometres

• $0.40/km for each additional kilometre

George has calculated the distance he will be driving. He will have to pay $164 to rent the car if the chooses Plan B.

How much will George have to pay to rent the car if he chooses Plan A?

Show all your work.

17 Two rival companies offer a delivery service for small parcels within the Montréal area. Both companies determine the cost of delivery according to the distance travelled to deliver a parcel.

Company A

Company A determines the cost of a delivery by using the rule $y_A = 0.10x + 4.50$.
    x: distance to be travelled to deliver a parcel, in km
    y_A: cost of delivery, in $

Company B

For Company B, the cost of a delivery varies directly with the distance to be travelled. The table of values below shows examples of delivery costs charged by Company B.

Distance to be travelled	Cost of delivery
8 km	$3.20
20 km	$8.00
30 km	$12.00

For what distance is the delivery cost the same for both companies?
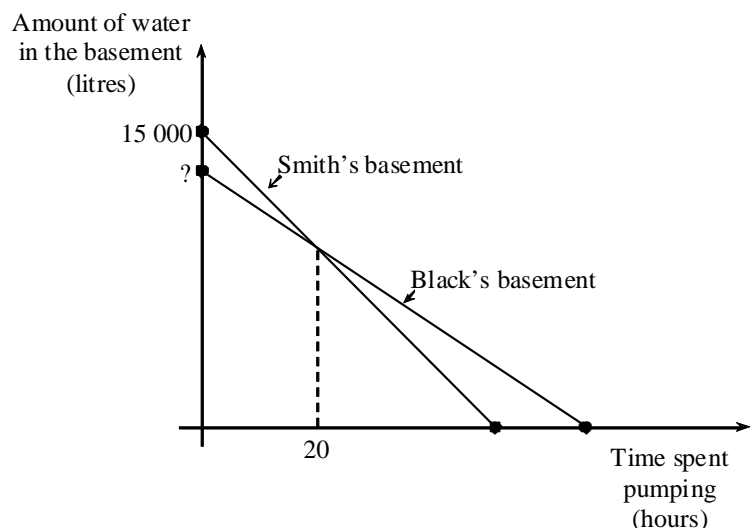Show all your work.

18 After a flood, pumps were used to remove water from the basements of the Smith and Black houses.

Before the pumping started, Smith's basement contained 15 000 litres of water. Smith's pump removed 300 litres of water per hour.

Black's pump removed 200 litres of water per hour.

Pumping began at the same time at both houses. After 20 hours of pumping, both basements contained the same amount of water.

The following graph represents the amount of water in the two basements according to the time elapsed from the moment the pumping began.

How much water, in litres, was in Black's basement before the pumping began?
Show all your work.

19 | The annual cost of participating in a squash club consists of a set membership fee plus an hourly rate for court use.

The table of values below represents the annual cost at this squash club depending on the hours of court use.

Number of hours	0	1	2	3	…
Annual Cost ($)	225	235	245	255	…

If Susan's bill for the year is $1025, how many hours did she play that year?

Show all your work.

20 | As part of a reforestation program, the Canadian government will plant 6000 trees.

Students are hired to plant the trees during the summer months.

The following table shows the relationship between the number of students hired, h, and the number of trees each student is expected to plant, t.
A minimum of 5 students will be hired.

h	5	10	15	20	25
t	1200	600	400	300	240

Draw the graph that represents this situation.

21 | Two schools opened in the same year each with a population of 300 students.
The Chateauguay School 's population increased at a rate of 130 students per year. The population of The Montreal School increased every year according to the rule:

$$y_M = 300(1.2)^x$$

where y_M = total population of The Montreal School
and x = the number of years since the school opened

On the 10th anniversary of its opening, which school had the greater population and by how many students?

| 22 | Jonathan and Ashley went on different business trips.
They both rented cars from the same dealer.

The total price charged included a fixed amount for the car rental, plus a specific charge for each kilometre driven.

Jonathan, who drove 600 km, had to pay $379.
Ashley drove 900 km and had to pay $544.
Carlo plans to go on a 1300-km trip.

How much would it cost Carlo to rent a car for a 1300-km trip from the same dealer?

| 23 | While Samantha, Jeremy and Ashley were on a school trip, they took many photographs with their digital cameras.

Samantha checked the cost of having her photos printed at the FOTOFLASH store near her house. This is the price list she picked up:

Number of photos	1 – 9	10 – 19	20 – 29	30 +
Cost per photo ($)	0.75	0.70	0.62	0.50

Jeremy and Ashley both had their photos printed at PICS Я US. This store charges a basic fee and a set cost for each photo printed.
Jeremy had 15 photos printed at a cost of $10.25. Ashley paid $19.25 to have 35 photos printed.
Samantha wants to have 28 photos printed.

How much will Samantha save if she has all 28 photos developed at the store that gives her the better deal?

| 24 | The mass of a 1000 kg radioactive substance decreases by 20% every year.
What will be the mass of the radioactive substance in four years?

| 25 | Environmentalists conducted a study of the pollution levels of Lake Hershey and Lake Morin. Lake Hershey's pollution level was increasing at a constant rate. Five years into the study, its pollution level was found to be 1600 parts per million. Three years later, its level was 2200 parts per million.

Lake Morin's pollution level is represented by the rule:

$$y_M = 16x^2$$

where x: is the number of years since the monitoring began
y: is the pollution level in parts per million.

Which lake was more polluted 14 years after the study began?

26 A parabola passes through the origin and the point (7, 88.2). An exponential function has an initial value of 40 and increases at a rate of 19.8% per year. If, for both functions, x is the number of years and $f(x)$ is the quantity in grams, which function will reach 200 grams first?

27 Given the following periodic function:
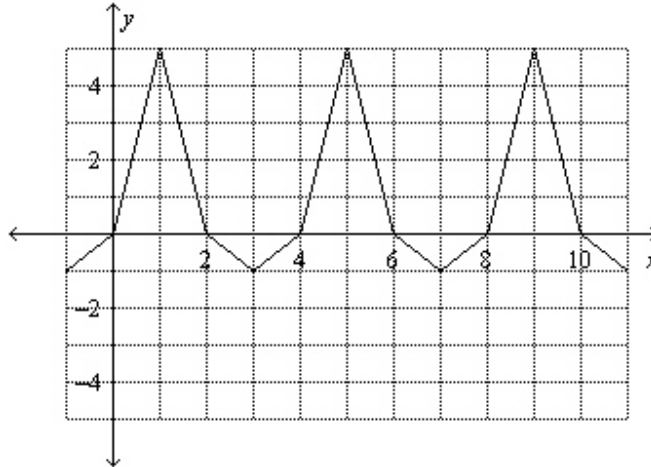
a) What is the period?
b) Determine $f(5)$
c) Determine $f(49.8)$

28 Given the following function:

$$f(x): \begin{cases} = 5x, \ 0 < x < 10 \\ = 4x + 9, \ 10 \le x < 100 \\ = 2.5x + 150, \ x \ge 100 \end{cases}$$

Evaluate: $f(3) + f(9.9) + f(10) + f(25) + f(100) + f(240)$

29 Given: $6x - 100y = 8400$ Determine the x and y intercepts.

30 Which of the following linear functions are increasing?

a) $6x - 9y = 8400$

b) $6x + 7y = 126$

c) $4y - 10x = 50$

d) $7x = 12 - 8y$

1

First offer
$400 \text{ m}^2 \div 40 \text{ m}^2 = 10$ times
$10 \times \$1 = \$10$
Second offer
$400 \text{ m}^2 \times 2.5$ cents $=$
$400 \text{ m}^2 \times \$0.025 = \$10$
Final answer   Neither one is a better deal because each worker would charge the same amount for a
400-square-metre lawn.

2

STATION 1	$1000 + 100x$
$\Rightarrow$	$5000 - 1000 = 4000$ $4000 \div 100 = 40$ 40 segments of 30 seconds
STATION 2	$2000 + 50x$
$\Rightarrow$	$5000 - 2000 = 3000$ $3000 \div 50 = 60$ 60 segments of 30 seconds
STATION 3	$150x$
$\Rightarrow$	$5000 \div 150 = 33.3$ 33.3 segments of 30 seconds

Final answer   The second radio station offers the best deal.

3

Let the initial price be $100.

	STORE 1	STORE 2

	10 % per day	10 % of the price announced the previous day
0	$100	$100
1	$90	$90 (10 % of 90)
2	$80	$81 (10 % of 81 = 8.10)
3	$70	$72.90 (10 % of 72.9 = 7.29)
4	$60	$65.61
	$60 (60 % of the initial price)	$65.61 (65.61 % of the initial price)

$$65.61 \% - 60 \% = 5.61 \%$$

Final answer : 5.61 %

4

Number of Hours Played	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cost at Annie's Favourite Place $	0.00	3.00	6.00	9.00	12.00	15.00	18.00	21.00	24.00	27.00	30.00
Cost at Gaby's Favourite Place $	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Cost at Eric's Favourite Place $	0.00	6.00	6.00	12.00	12.00	18.00	18.00	24.00	24.00	30.00	30.00

Answer   Annie's favourite place offers the best deal because, in comparison to the others, a game is less expensive if it lasts a fraction of an hour.

5

Time (in h)	Type A bacteria	Type B bacteria
0	1000	500
1	2000	1500
2	4000	4500
3	8000	13 500
4	16 000	40 500
5	32 000	121 500

Answer   Type B bacteria would be more numerous.

6   The graph of the step function indicates that it will cost $60 to travel between 200 and 250 km.

Rule of correspondence of the partial variation function
Let d: distance travelled in km
Let c: rental cost in $
rate of change $= \dfrac{70 - 40}{200 - 0} = \$0.15/\text{km}$
fixed cost $= \$40$ $\qquad\qquad c = 0.15d + 40$

Rental cost for 240 km: $0.15 \times 240 + 40 = \$76$
Difference in cost: $\$76 - \$60 = \$16$
Answer   The exact difference in cost between the two plans is $16.

7 Table of values

Number of days	M_1 ($)	M_2 ($)
0	0.01	1.00
1	0.03	2.00
2	0.09	4.00
3	0.27	8.00
4	0.81	16.00
5	2.43	32.00

The difference between the two amounts

$M_2 - M_1 = 32.00 - 2.43 = \$29.57$

Answer: The difference between the two amounts is $29.57.

8 **Company A**
Rule of correspondence is $C = 20 + 0.4x$
By substitution $C = 20 + 0.4(280) = 132$
Cost of 280 km is $132

Company B
Flat rate of $115

Company C
The next interval in the table would be a cost of $105

Most expensive is Company A
Least expensive is Company C

Answer   Difference in cost is $132 − $105 = $27.

9

Number of years passed	Value of the sport car ($)	Value of the truck ($)
0	60 000	40 000
1	48 000	36 000
2	38 400	32 400
3	30 720	29 160
4	24 576	26 244
5	19 660.80	23 619.60

Answer: The vehicle that will be worth the most is the truck.

10   Gains with company A

Month	0	1	2	…	12
Sum ($)	100	105	110.25	…	179.59

After 1 year, the shares in company A are worth $179.59.

Gains with company B

From the table of values, the shares of company B are worth $58.85.

Total gains

$179.59 + 58.85 = 238.44$

Profit

$238.44 - (100 + 75) = 63.44$

Answer: After 12 months, her profit is $63.44.

11   Table of Values

Hours Quantity of litres	10	11	12	13	14	15	16	17
Pump A	0	2000	4000	6000	8000	10 000	12 000	14 000
Pump B	0	0	3000	6000	9000	12 000	15 000	18 000
Total	0	2000	7000	12 000	17 000	22 000	27 000	32 000

Answer: The pool will be filled to capacity at 5:00 p.m.

12   Amount charged by store A

$$y_A = 4(1.15)^7 = \$10.64$$

Amount charged by store B

Rate of change: $\dfrac{7.75 - 4.75}{5 - 1} = 0.75$

Number of days after the due date	Total amount charged
5	$7.75
6	$7.75 + \$0.75 = \$8.50$
7	$8.50 + \$0.75 = \$9.25$

Difference between the two amounts

$10.64 - 9.25 = \$1.39$

Answer: The difference between the total amounts charged by these stores for a movie returned 7 days after the due date is $1.39.

13 Rental cost for the **1st** company

Find the rate of change *a:*

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{80 - 60}{200 - 0}$$ Rule $y = 0.1x + 60$

$$a = 0.1$$

Cost for 120 km $y = 0.1 \times 120 + 60$

$y = 72$

Rental cost for the **2nd** company

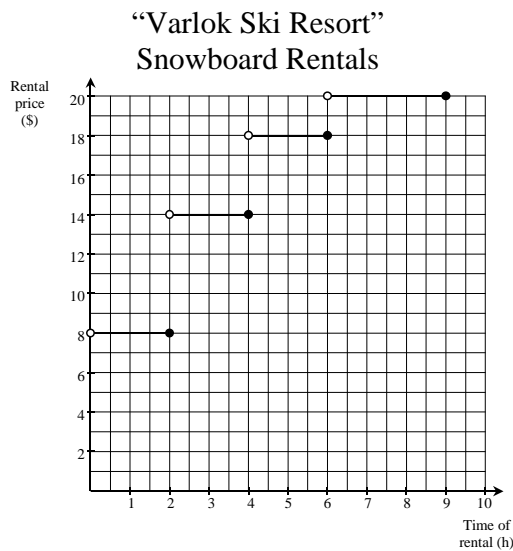| Distance (km)	]80, 120]	]120, 160]
Cost ($)	60	80

Cost for **120** km: $**60**

The difference between the rental costs for a distance of 120 km

$72 – $60 = $12

Answer   The difference in the rental costs for a distance of 120 km is $12.

14

Time of rental (hours)	Total cost ($)
]0, 2]	8
]2, 4]	14
]4, 6]	18
]6, 9]	20

"Varlok Ski Resort"
Snowboard Rentals

Five criteria to consider:
1.   The overall graph is correct.
2.   The axes are identified correctly.
3.   The axes are scaled correctly.
4.   The last segment ends after 9 hours.
5.   The segments are open on the left.

15

Country A	
Year	Number of people infected with AIDS
2005	1100
2006	1166
2007	1236
2008	1310
2009	1389
2010	1472

Country B	
Year	Number of people infected with AIDS
2005	800
2006	888
2007	986
2008	1094
2009	1215
2010	1348

Answer:　In 5 years, there will be more people infected with AIDS in Country **A**.

16 • Distance George will drive
Cost under Plan B
　　$164
Cost for the first 120 kilometres under Plan B
　　120 km × $0.50/km = $60
Cost for additional kilometres
　　$164 − $60 = $104
Number of additional kilometres
　　$104 ÷ $0.40/km = 260 km
Number of kilometres George will drive
　　120 km + 260 km = 380 km

Cost of rental under Plan A to travel 380 km

According to the graph, we can say that the rental cost will be $120 for a distance of 380 km.

Answer:　If George chooses Plan A, he will have to pay $**120** to rent the car.

17 • Rule representing the cost of a delivery carried out by Company B

Rate of change

$$\frac{\$3.20 - \$8.00}{8 \text{ km} - 20 \text{ km}} = \frac{-\$4.80}{-12 \text{ km}}$$
$$= \$0.40/\text{km}$$

Since the delivery cost is proportional to the distance travelled
$$y_B = 0.40x$$

x: distance travelled to deliver a parcel, in km
y_B: cost of delivery carried out by Company B, in \$

• Distance for which the delivery cost is the same for both companies

We are looking for the value of x for which $y_A = y_B$.

x	$y_A = 0.10x + 4.50$	$y_B = 0.40x$
0	4.5	0
20	6.5	8
18	6.3	7.2
15	6	6

The delivery cost for a distance of 15 km is the same for both companies.

Answer: The delivery cost for a distance of **15** km is the same for both companies.

18 • Amount of water in the two basements 20 hours after the pumping began

$$y_S = 15\,000 - 300x$$

x: time elapsed from the moment the pumping began, in hours
y_S: amount of water in Smith's basement, in litres

After 20 hours spent pumping
$$y_S = 15\,000 - 300(20)$$
$$= 9000$$

After 20 hours spent pumping, the two basements each contained 9000 litres of water.
• Number of litres of water in Black's basement before the pumping began
20 hours after the pumping began, there were still 9000 litres of water
In 20 hours, the pump remove 20×200 litres = 4000 litres of water
Number of litres of water before the pumping began
$$9000 \text{ L} + 4000 \text{ L} = 13\,000 \text{ L}$$
Answer: There were **13 000** litres of water in Black's basement before the pumping began.
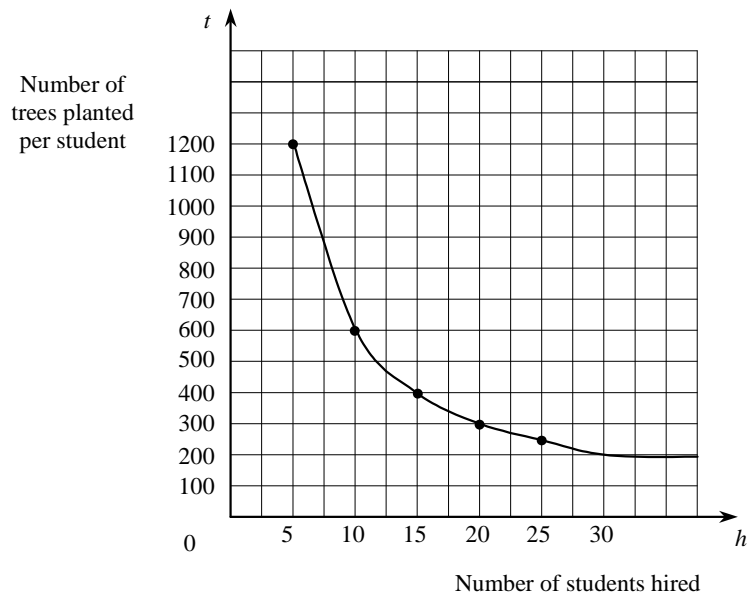
19

y : annual cost
x : number of hours

$$y = 10x + 225$$

$$1025 = 10x + 225$$

$$x = 80$$

Answer   Susan played 80 hours.

20

Number of trees planted per student

Number of students hired

The following criteria must be satisfied:
1) Graph correctly represents the relation
2) Axes labelled correctly
3) Axes scaled appropriately
4) Curved graph with or without arrows; no heavy dots at the end

21

The Montreal School

$$y = 300(1.2)^x$$
$$= 300(1.2)^{10}$$
$$= 1857.52\ldots$$

The Chateauguay School

$$y = 300 + 130x$$
$$y = 300 + 130(10)$$
$$= 300 + 1300$$
$$= 1600$$

Difference
$$1857 \text{ or } 1858 - 1600 = 257 \text{ or } 258$$

Answer: On its 10th anniversary, The Montreal School had the greater population by 257 (or 258) students.

22 x : number of km driven
y : cost to rent in dollars

Rate of variation (cost per km)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{544 - 379}{900 - 600}$$
$$= 0.55$$

Equation representing rental cost

$$y = .55x + b$$
$$544 = .55(900) + b$$
$$544 = 495 + b$$

Fixed cost $= \$49$

$$\therefore y = .55x + 49$$
$$y = .55(1300) + 49$$
$$y = 715 + 49$$
$$y = \$764$$

Answer: It would cost Carlo $\$\mathbf{764}$ to rent a car for a 1300-km trip.

23   Determine the linear relation defining the cost of printing photos at PICS Я US $y = ax + b$
Cost per photo

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{19.25 - 10.25}{35 - 15}$$

$$a = \frac{9}{20}$$

$$a = 0.45$$

Initial value

$$y = ax + b$$
$$10.25 = 0.45(15) + b$$
$$b = 3.50$$
$$y = 0.45x + 3.50$$

Determine the cost of printing 28 photos at PICS Я US

$$= 0.45(28) + 3.50$$
$$= \$16.10$$

Determine the cost of printing 28 photos at FOTOFLASH

$$= 0.62 \times 28$$
$$= 17.36$$

Determine the difference between the two shops

$$\$17.36 - \$16.10 = \$1.26$$

Answer: Samantha will save $\$\mathbf{1.26}$ by having her photos developed at the store that gives her the better deal.

24

x (number of years)	y (remaining mass)
0	1000
1	800
2	640
3	512
4	409.6

Answer: In 4 years, the mass of the radioactive substance will be **409.6** kg.

25   Rate of change for Lake Hershey

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{2200 - 1600}{8 - 5}$$

$$= \frac{600}{3}$$

$$= 200$$

Initial value

$$y = ax + b$$
$$y = 200x + b$$
$$1600 = 200(5) + b$$
$$1600 - 1000 = b$$
$$600 = b$$
$$y = 200x + 600$$

Lake Morin

$$y = 16x^2$$
$$= 16(14)^2$$
$$= \mathbf{3136}$$

Lake Hershey

$$200(14) + 600 = \mathbf{3400}$$

Answer: **Lake Hershey** was more polluted, 14 years after the study began.

26

	Parabola	Exponential
	y=1.8x^2	y=40(1.198)^x
0	0	40
1	1.8	47.92
2	7.2	57.40816
3	16.2	68.77497568
4	28.8	82.39242086
5	45	98.7061202
6	64.8	118.249932
7	88.2	141.6634185
8	115.2	169.7127754
9	145.8	203.3159049
10	180	243.5724541
11	217.8	291.7998

The exponential function will reach 200 grams first.

27

a) Period $= 4$

b) $f(5) = 5$

c) $f(49.8) = f(1.8)$ equation of line from $x = 1$ to $x = 2$ is $y = -5x + 10$

so, $f(1.8) = -5(1.8) + 10$

$f(1.8) = 1$

28 $f(3) = 15$

$f(9.9) = 49.5$

$f(10) = 49$

$f(25) = 109$

$f(100) = 400$

$f(240) = 750$ **Total: 1372.5**

29 x-intercept: $x = 1400$, y-intercept: $y = -84$

30 a and c are increasing