Math 4CST

St. Thomas High School

MIDTERM REVIEW PACKAGE

Name: _____

Teacher: _____

Exam Date & Time: _____

<u>Materials to bring</u>: (in a clear ziplock bag)

- Pencils, Eraser, Ruler, Sharpener, Highlighters
- **CALCULATOR**
- **MEMORY AID**: one 8 ½ by 11 page, hand-written, with **YOUR NAME** in the top right corner

BREAKDOWN OF THE EXAM:

- 6 Multiple Choice (4 marks each)

- 4 Short Answer (4 marks each – no part marks)

- 6 Long Answer (10 marks each)

Topics:

Functions

- Properties of Functions (Domain, Range, Variation, Signs, x and y-intercepts)
- Linear Functions
- Quadratic Functions
- Step Functions
- Piecewise Functions
- Periodic Functions
- Exponential Functions

Systems of Equations

- Solving by Comparison
- Solving by Substitution
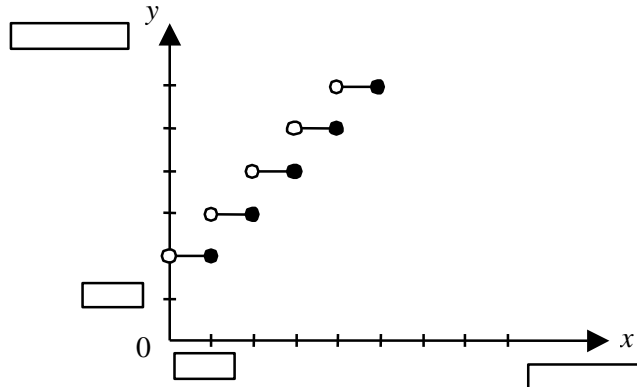- Solving by Elimination
- Word Problems

Functions

Multiple Choice Questions

1 A long distance call between two particular cities costs $1.00 for the first minute. Each additional minute costs $0.50.

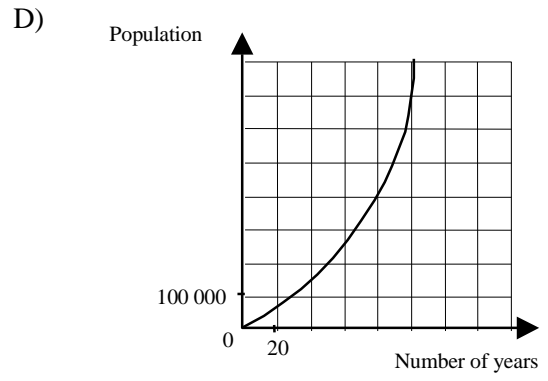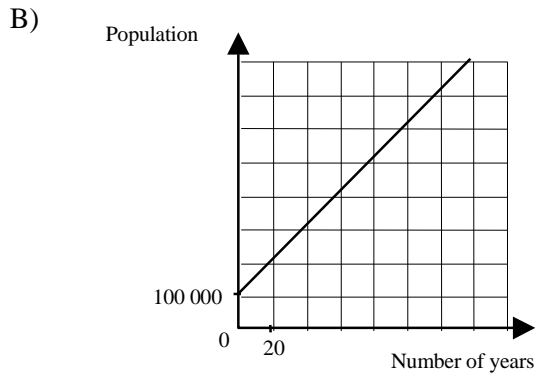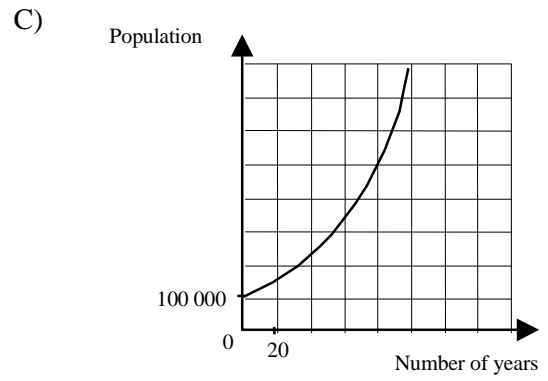The step graph below represents this situation.

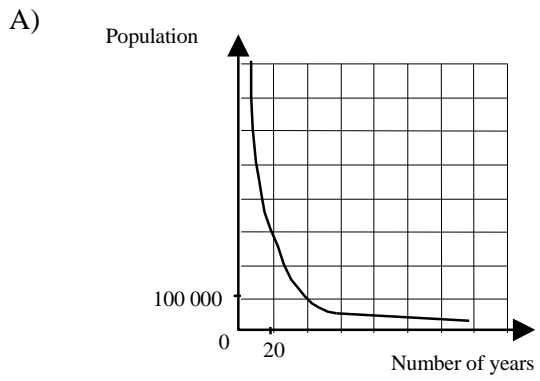From the given information, which of the following choices represents the dependent variable, the independent variable and the appropriate scale for each axis ?

	Dependent Variable	Independent Variable	Scale	
			x-axis	y-axis
A)	LENGTH	COST	1	50
B)	COST	LENGTH	50	1
C)	LENGTH	COST	50	1
D)	COST	LENGTH	1	50

2 The population of a city of 100 000 increases by 2% each year. A study was made of the relationship between the number of years elapsed and the population growth of the city.

Which of the graphs below represents this situation?

A)

Population

100 000

0   20

Number of years

C)

Population

100 000

0   20

Number of years

B)

Population

100 000

0   20

Number of years

D)

Population

100 000

0   20

Number of years

3   In 1990, a town had a population of 100 000. Since 1990, the population of this town has decreased by 10% per year.

Which of the following graphs shows the relationship between population and the number of years that have passed since 1990?

A)

B)

C)

D)
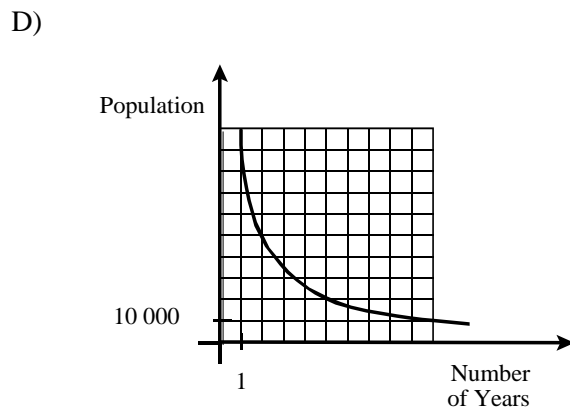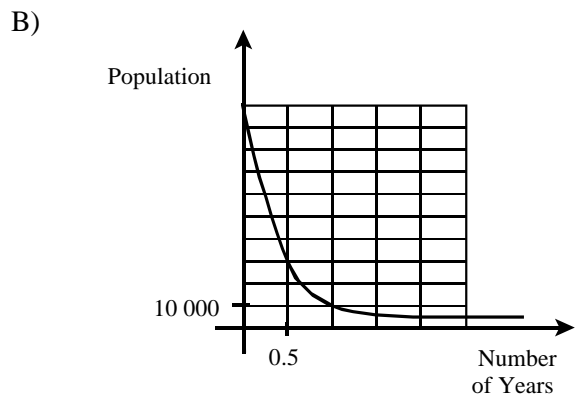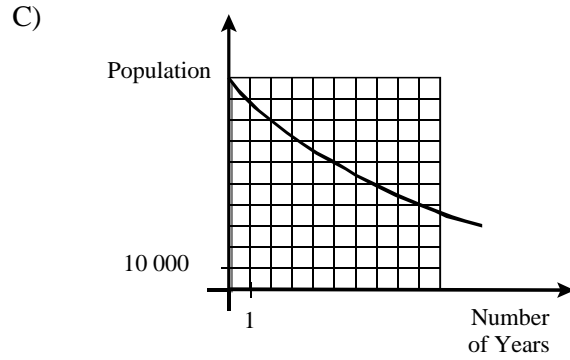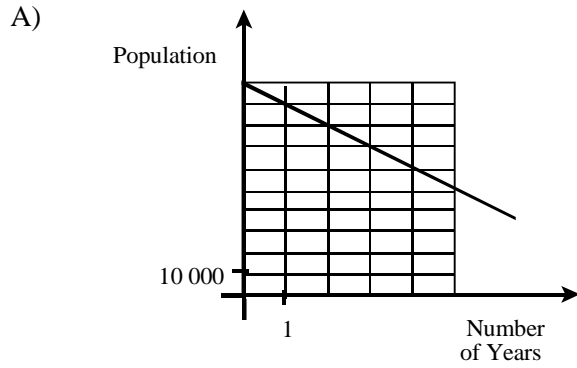
4 Yannick's parents plan to start giving him an allowance on his tenth birthday. He will get $4 a week and this amount will be increased by $3 every two years. This information is given in the table of values below.

Yannick's age	10	11	12	13	14	15	16	17
Amount of allowance ($)	4	4	7	7	10	10	13	13

Which graph best represents this situation?

A)

B)

C)

D)
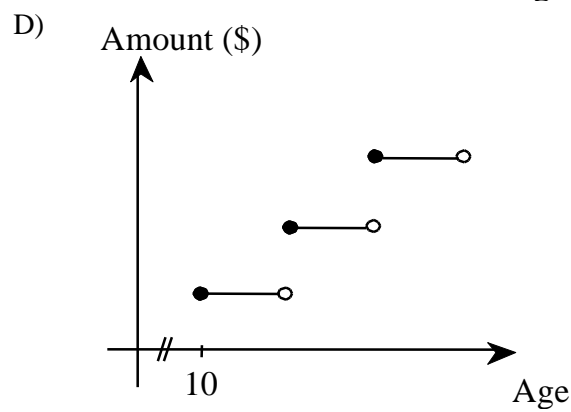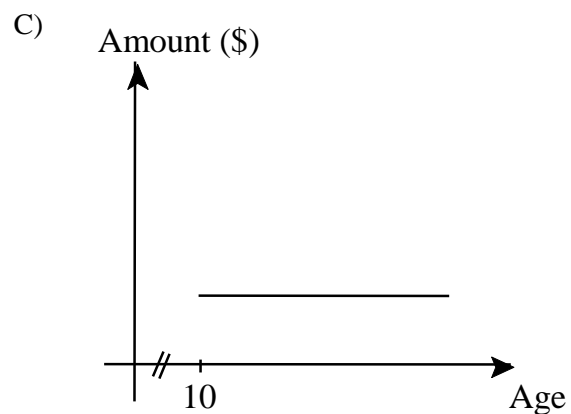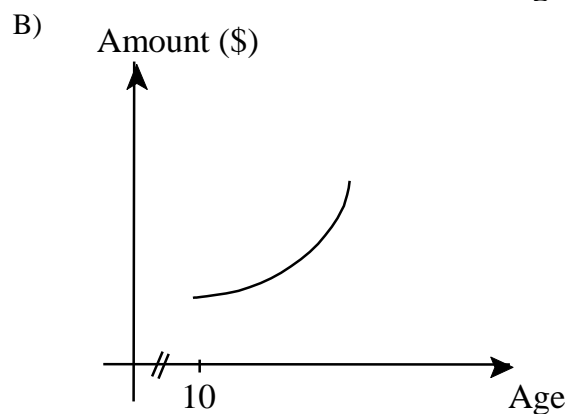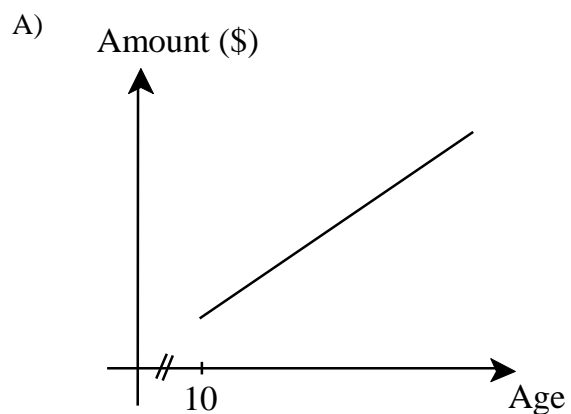
5 After comparing car rental costs between two companies, Mr. Kelly constructed the graph below.

Car-rental Costs
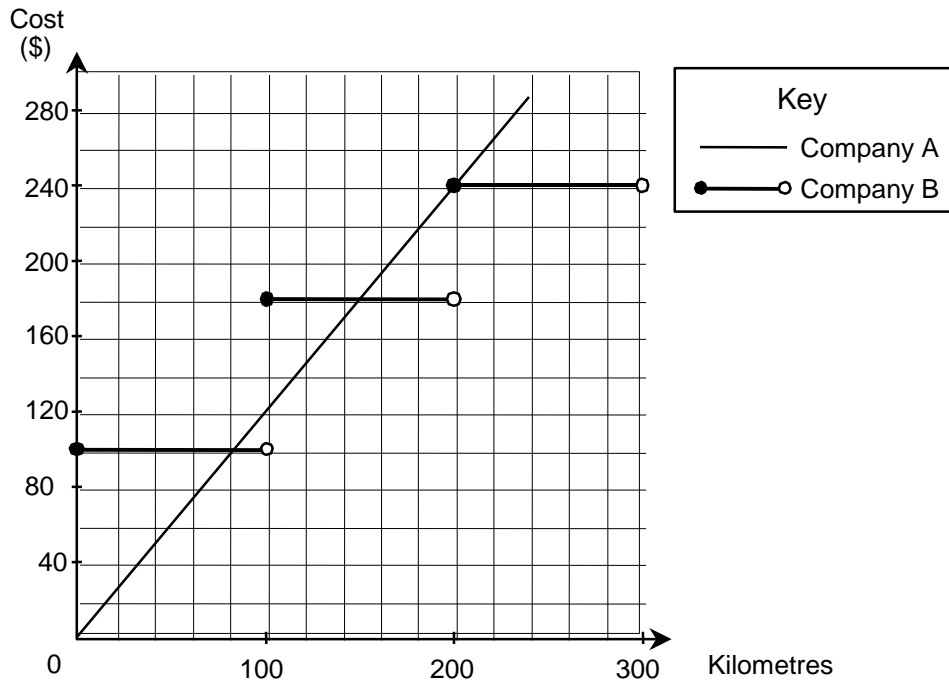
In analyzing the graph, he made the following conclusions:

1- For 100 km, company B charges less than company A.
2- For 100 km, the difference in car-rental costs between company A and company B is $60.
3- For 200 km, both companies charge the same amount.

Which of Mr. Kelly's conclusions are TRUE?

A)   1 and 2 only   C)   2 and 3 only

B)   1 and 3 only   D)   1, 2 and 3

6 | The cost of mailing a parcel depends on its mass.

Cost of Mailing a Parcel

The graph on the right represents this situation.
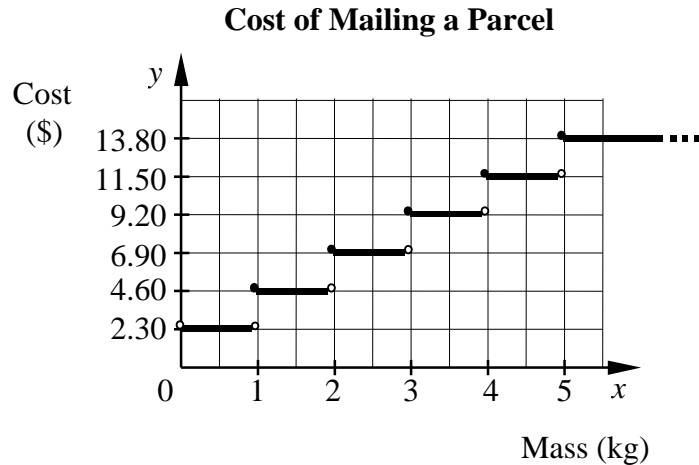
The cost of mailing a parcel in relation to its mass is detailed below.

1. The cost of mailing a one-kilogram parcel is $2.30.
2. All parcels weighing over 5 kg are mailed at a fixed cost.
3. There is no cost for mailing a parcel that weighs 0.25 kg.
4. The cost of sending a 5-kg parcel is twice as expensive as sending a 2.5-kg parcel.
5. It costs $6.90 to mail a parcel weighing less than 3 kg.

Which of the statements above are true?

A)   1, 2 and 4

B)   2, 3 and 5

C)   2 and 4

D)   4 and 5

Short Answer Questions

7 | A store manager is hiring sales people. During the interviews, he gives each candidate three different options of weekly pay.

Given that **n** is the number of sales and **p** is the amount of pay in dollars, the following graph shows the three options offered to the candidates:

Which option of weekly pay would encourage the greatest number of sales?
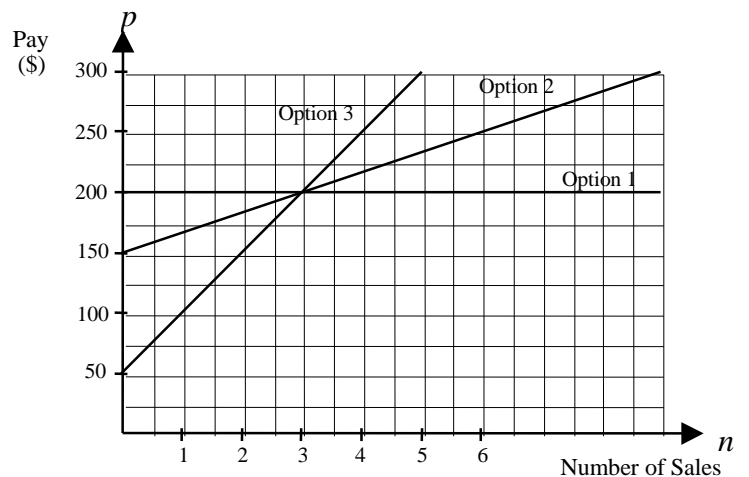
8   Mary would like a hook up for the Internet.
She signs up with the "NetPlus" provider.
The monthly fees for a first-time user are listed below.

Number of hours on line	Fees ($)
[0, 7]	10.00
]7, 8]	11.50
]8, 9]	13.00
]9, 10]	14.50
]10, 11]	16.00

Draw the graph of this situation.

9   John has a window-washing business. For the first hour or part thereof he charges $12, which includes his travelling expenses. Thereafter, he charges $4 for every additional hour or any part thereof for his labour.

John never takes more than 3 hours washing windows for any particular customer.

Draw a graph which represents this situation.

10   Upon exposure to the sun, a 10-kg block of ice loses 30% of its mass for each minute that elapses.

Draw the graph showing the variation of the mass of the block of ice for the first five minutes that it is exposed to the sun.

11   At the end of 1999, Paul bought a new car for $25 000. Each subsequent year the car lost 18% of its previous year's value.

Consider the relation between the value of the car and the number of years since the car was bought.

How much was the car worth at the end of 2002?

12 Placing an ad in *The Frontenac Herald* costs $6 for the first 20 words plus $0.40 for each additional word. *The Tribune*, on the other hand, charges $12.00 for the first 20 words plus $0.16 for each additional word.

Consider the cost of the ads in relation to the number of additional words.
The cost of the ads is represented by the following system of linear relations.

$$y_1 = 6 + 0.4x$$
$$y_2 = 12 + 0.16x$$

Draw a graph to represent this system and indicate the solution on the graph.

13 The following graph shows the cost of a day ticket to an amusement park according to a person's age.
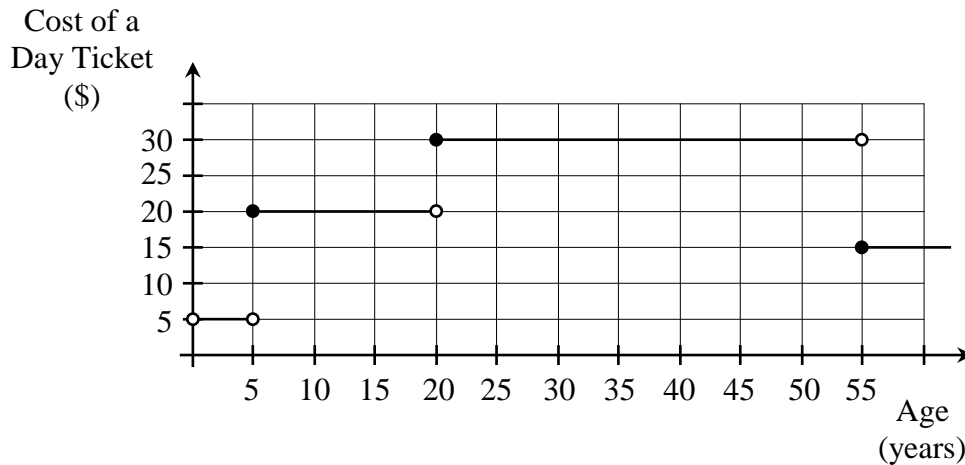
Helen and Dennis will spend a day at this park with their two children. Helen, Dennis and their children are 52, 55, 15 and 20 years of age, respectively.

What will be the total cost of the day tickets for this family of 4?

14 A company that has been developing computer processors since 1988 estimates that the speed of a processor increases by 21% every year compared with the previous year. On January 1, 1988, the speed of a processor was 66 megahertz (MHz).

This situation is represented by the rule $y = 66(1.21)^x$.

x: time that has passed since January 1, 1988, in years
y: speed of a processor, in MHz

At this rate, what will the speed of a processor be on January 1, 2010, to the nearest MHz?

15 A youth club in Orbite is organizing Canada Day events. The members of the club must distribute information pamphlets to each of the 2500 residences of the village. The table below represents the relation between the number of youth club volunteers and the number of houses that each volunteer must visit.

where x: represents the number of volunteers
         y: represents the number of houses

x	5	10	15	20	25
y					

a)   Complete the table of values in the *Answer Booklet*.

b)   What type of relation is represented?

Application Questions

16 Nancy invested $100 in company A and $75 in company B. After a year, she checked to see how her investments were doing.

- The value of her shares in company A increased at a rate of 5% a month.
- The value of her shares in company B decreased according to the pattern below:

Month	0	3	6	9	12
Amount ($)	75	70.59	66.44	62.53	58.85

After twelve months, what was the total profit that Nancy earned from her two investments?

17 Peter wants to rent a car. He checks the rental costs at two car rental companies. The graph below shows the relation between the rental cost of each company and the distance travelled. Peter plans to drive 120 km.

The Cost of Renting a Car
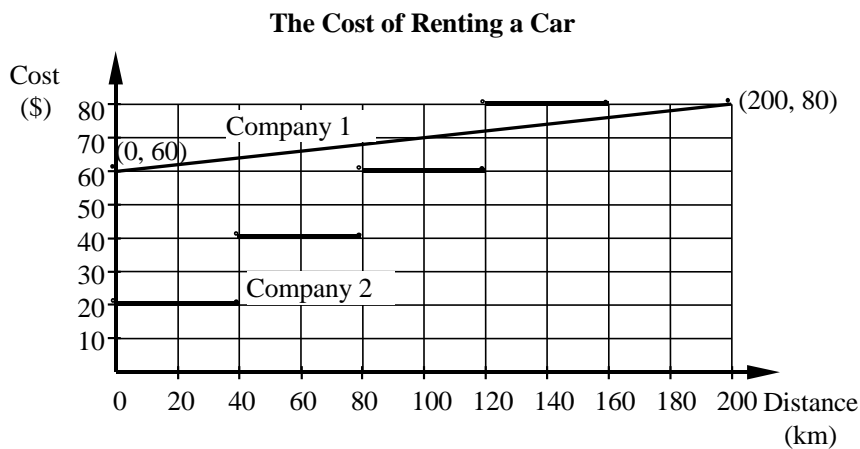
What is the difference in the rental cost between the two companies?

18 | In 2005, a doctor compared the number of people infected with AIDS in two African countries. In Country A, 1100 people had AIDS. This number is expected to increase by 6% per year. In Country B, 800 people were infected with AIDS. This number is expected to increase by 11% per year.

In 5 years, which country will have more people infected with AIDS?

19 | George wants to rent a car for a trip. The car rental company asks him to choose one of the following two rate plans.

Plan A
Under Plan A, the rental cost based on distance travelled is represented by the following graph.
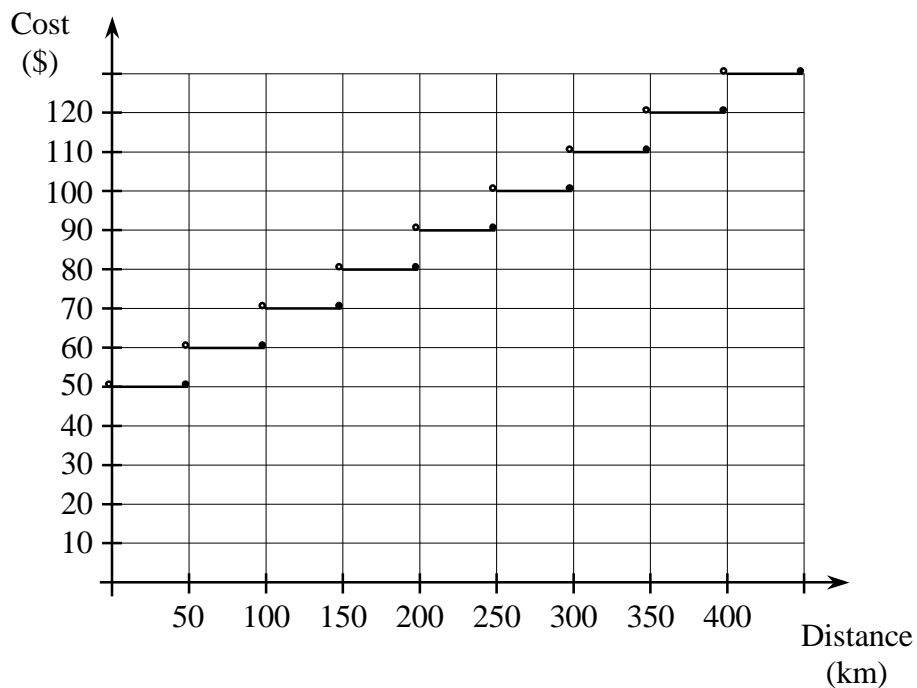
Plan B

Under Plan B, the rental cost is determined as follows:

- $0.50/km for the first 120 kilometres
- $0.40/km for each additional kilometre

George has calculated the distance he will be driving. He will have to pay $164 to rent the car if the chooses Plan B.

How much will George have to pay to rent the car if he chooses Plan A?

20 | Alex wants to rent a moving truck for a few hours. Two companies offer the service he needs.

Company A charges the following rates to rent its trucks:

- $60 for the first hour or portion of an hour
- $20 for each additional hour or portion of an hour

The following graph shows the cost of renting from company B in relation to the amount of time for which the truck is rented.
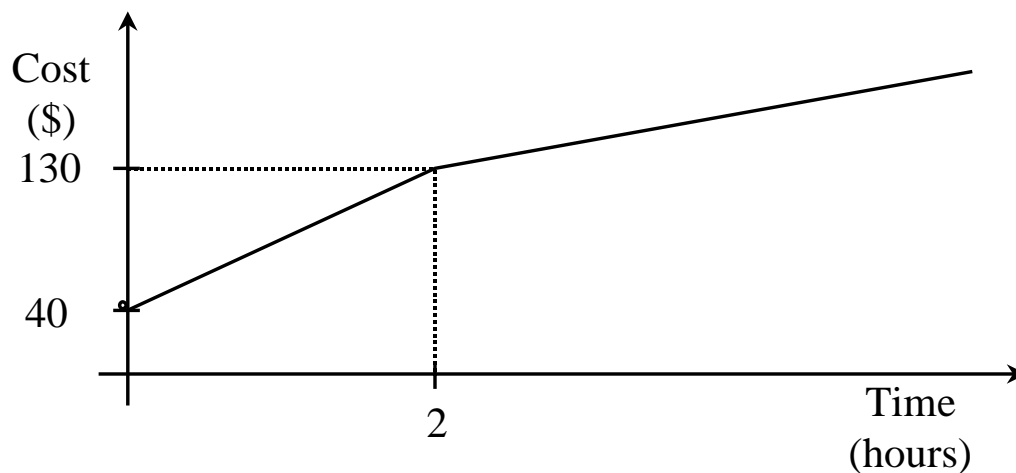
Alex decided to rent a truck from company A for 4.5 hours. If he had chosen company B, he would have paid $25 more.

After the 2nd hour, how much does company B charge for each additional hour?

21 Stephan and Emilie are each trying to save $2000. Stephan's plan is to put $1000 into a savings account that will earn 8% per year. His money will accumulate according to the rule:

$$y = 1000(1.08)^x$$

where x: is the number of years since his initial deposit
               y: is the amount of money accumulated.

Emilie is putting money into her piggy bank according to the following graph:

Emilie's Accumulated Savings
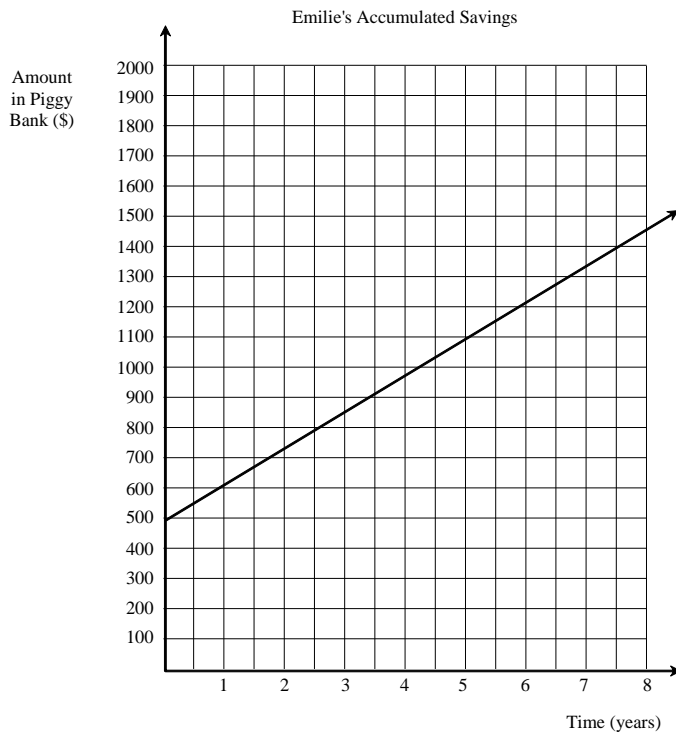
Who will be the first to have saved $2000?

22.   Given the function $f(x) = \begin{cases} 3 & , if\ x: [0,8[\\ -2x + 4 & , if\ x: [8,12[\\ -x^2 & , if\ x: [12,20] \end{cases}$, determine which of the following is false?

A) the function is positive over [0,8[B) the function is decreasing over [8,20]
C) the function has an initial value of 3   D) the function has a zero

23.   The graph below shows the flight path of two stunt planes.

Height of Stunt Planes

Height of Planes
(m)

Plane 1

Plane 2
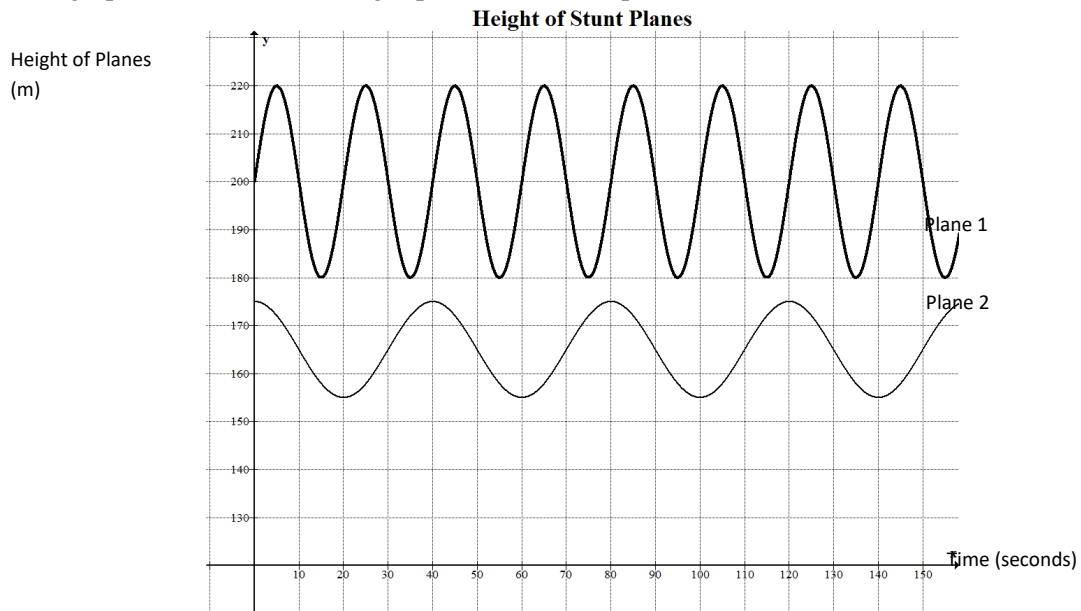
Time (seconds)

What is the distance between the planes after 190 seconds?

24.   A stone is dropped into a wishing well. The distance the stone has fallen over time can be represented by a second-degree polynomial function. Two seconds after the stone was dropped it has fallen 20 m. Determine how deep the well is if it takes the stone 6 seconds to hit the bottom.

25. A car rental company charges customers based on the distance the car travels. The amount charged is shown in the graph below.
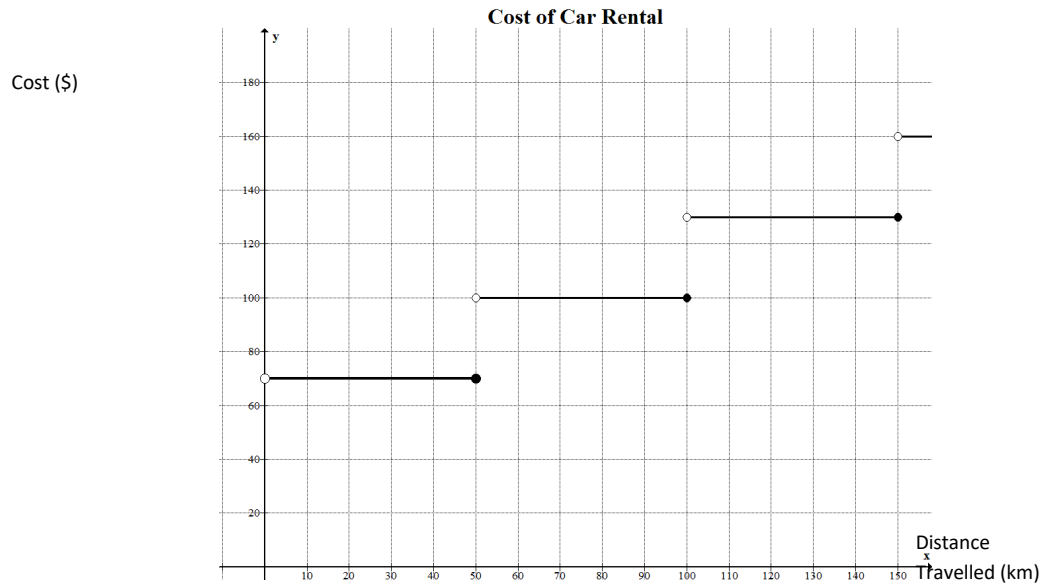
Cost of Car Rental

Cost ($)

Distance Travelled (km)

a) What is the furthest a customer can travel if they are willing to pay up to $140 and how much would it cost?

b) Determine the cost of renting the car if a customer travels 260 km.

26. The weekly salary of an employee is based on the amount of sales and is given by the rule:

$$f(x) = \begin{cases} 300 & 0 \leq x \leq 1500 \\ 0.1x + 300 & 1500 < x \leq 4000 \\ 0.15x + 100 & x > 4000 \end{cases}$$ where $f(x)$ is the weekly salary and x is the amount of sales the employee made.

Last month, an employee had $1200 worth of sales in the first week, $2000 in sales in the second week, and $4000 in sales in the third week.

At the end of the month the employee had earned a salary of $2100.

What was the employee's amount of sales in the fourth week of the month?

27.   The populations of two towns are being studied. Today, the population of Smallville is 25 000 and its population is increasing by 10% every year.
The table below shows the population of Bigtown since 2006.

Time (in years since 2006)	Population
0	40 000
1	41 600
2	43 264

Determine which town will be the first to have a population of 50 000.

28   A restaurant owner must decide which of three radio stations will be given his publicity campaign. Each of these stations has the same number of listeners. The restaurant owner has $5000 to spend on publicity.

-    The first station charges $1000 for the advertisement plus $100 for every 30 seconds on the air.
-    The second station charges $2000 for the advertisement plus $50 for every 30 seconds on the air.
-    The third station doesn't charge for the advertisement but charges $150 for every 30 seconds on the air.

Which radio station should the restaurant owner choose if he wants the most air time for his money?

29   Two stores decide to liquidate a product that they normally sell for the same price. The first advertises that, from the first day of the liquidation sale, it will take 10% off the initial price each day until a discount of 50 % is reached. At the second store, each day 10 % will be taken off the price announced the previous day until a discount of 50% is reached. The liquidation sales start on the same day.

What is the difference in per cent between the sale prices of the product in the two stores on the 4th day of the liquidation sale?

30 | Annie, Gaby and Eric love to play pool. They play in various pool halls in town.

The cost of playing at Annie's favourite pool hall is represented by the adjacent graph.
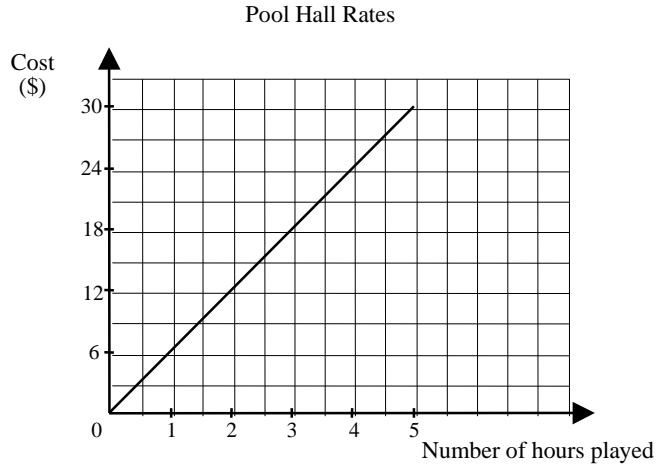
Pool Hall Rates

The cost of playing at Gaby's favourite pool hall is shown on the table of values below:

Pool Hall Rates

Number of hours played	0	1	2	3	4	5
Cost $	15.00	15.00	15.00	15.00	15.00	15.00

At Eric's favourite place, they charge a flat rate of $6.00 per table per hour.

If a game lasts less that two and a half hours, whose favourite pool hall offers the best deal?

Justify your answer.

31 A car leasing company has two rental plans.

The following graph shows the relationship between the distance travelled and the cost of renting a car under each plan.
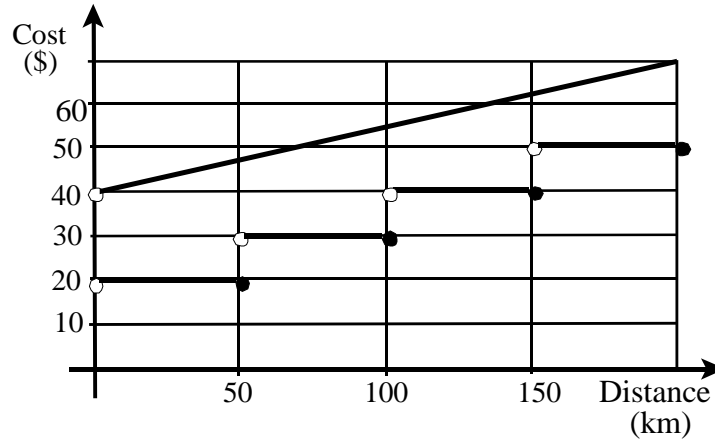
A customer drives 240 km in a rented car.

What is the exact difference in cost between the two plans?

32 The following brainteaser came up during a t.v. quiz show.

        M_1:   An amount of $0.01 triples every day.

        M_2:   An amount of $1.00 doubles every day.

After 5 days, what will be the difference between the two amounts?

33 In two video rental stores, customers must pay a fine if they return a movie after the due date.

To determine the total amount charged for renting a movie when it is returned after the due date, store A uses the following rule:

$$y_A = 4(1.15)^x$$

Where y_A represents the total amount charged, in dollars, for renting a movie and x represents the number of days after the due date.

Store B charges a flat rate for renting a movie plus a fine for each day after the due date. The following table of values gives examples of the total amounts charged by store B.

Number of days after the due date	Total amount charged
1	$4.75
5	$7.75
10	$11.50

What is the difference between the total amounts charged by these stores for a movie returned 7 days after the due date?

34 The Varlok Ski Resort rents snowboards.

Rental prices vary according to the number of hours the snowboard is rented.

- two hours or less: $8
- More than 2 hours but less than or equal to 4 hours: $14
- More than 4 hours but less than or equal to 6 hours: $18
- More than 6 hours but less than or equal to 9 hours: $20

Construct a graph that represents this situation. (Be sure to label the axes.)

35 Nancy invested $100 in company A and $75 in company B. After a year, she checked to see how her investments were doing.

- The value of her shares in company A increased at a rate of 5% a month.

- The value of her shares in company B decreased according to the pattern below:

Month	0	3	6	9	12
Amount ($)	75	70.59	66.44	62.53	58.85

After twelve months, what was the total profit that Nancy earned from her two investments?

36 Two rival companies offer a delivery service for small parcels within the Montréal area. Both companies determine the cost of delivery according to the distance travelled to deliver a parcel.

Company A
Company A determines the cost of a delivery by using the rule $y_A = 0.10x + 4.50$.
  x: distance to be travelled to deliver a parcel, in km
  y_A: cost of delivery, in $

Company B
For Company B, the cost of a delivery varies directly with the distance to be travelled. The table of values below shows examples of delivery costs charged by Company B.

Distance to be travelled	Cost of delivery
8 km	$3.20
20 km	$8.00
30 km	$12.00

For what distance is the delivery cost the same for both companies?

37 After a flood, pumps were used to remove water from the basements of the Smith and Black houses.

Before the pumping started, Smith's basement contained 15 000 litres of water. Smith's pump removed 300 litres of water per hour.

Black's pump removed 200 litres of water per hour.

Pumping began at the same time at both houses. After 20 hours of pumping, both basements contained the same amount of water.

The following graph represents the amount of water in the two basements according to the time elapsed from the moment the pumping began.
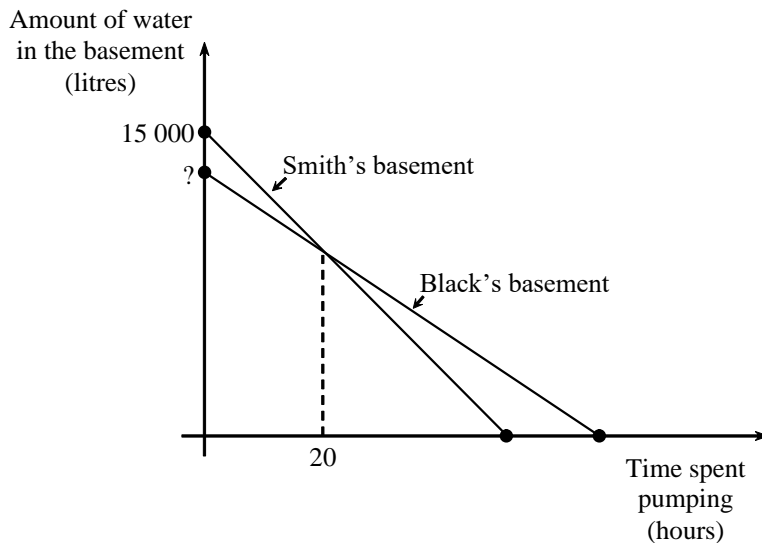
How much water, in litres, was in Black's basement before the pumping began?

38 Jonathan and Ashley went on different business trips.
    They both rented cars from the same dealer.

    The total price charged included a fixed amount for the car rental, plus a specific charge for each kilometre driven.

    Jonathan, who drove 600 km, had to pay $379.
    Ashley drove 900 km and had to pay $544.
    Carlo plans to go on a 1300-km trip.

    How much would it cost Carlo to rent a car for a 1300-km trip from the same dealer?

39 While Samantha, Jeremy and Ashley were on a school trip, they took many photographs with their digital cameras.

    Samantha checked the cost of having her photos printed at the FOTOFLASH store near her house. This is the price list she picked up:

Number of photos	1 – 9	10 – 19	20 – 29	30 +
Cost per photo ($)	0.75	0.70	0.62	0.50

    Jeremy and Ashley both had their photos printed at PICS Я US. This store charges a basic fee and a set cost for each photo printed.
    Jeremy had 15 photos printed at a cost of $10.25. Ashley paid $19.25 to have 35 photos printed. Samantha wants to have 28 photos printed.

    How much will Samantha save if she has all 28 photos developed at the store that gives her the better deal?

40 The mass of a 1000 kg radioactive substance decreases by 20% every year.
    What will be the mass of the radioactive substance in four years?

41 Environmentalists conducted a study of the pollution levels of Lake Hershey and Lake Morin. Lake Hershey's pollution level was increasing at a constant rate. Five years into the study, its pollution level was found to be 1600 parts per million. Three years later, its level was 2200 parts per million.

Lake Morin's pollution level is represented by the rule:

$$y_M = 16x^2$$

where x: is the number of years since the monitoring began
                y: is the pollution level in parts per million.

Which lake was more polluted 14 years after the study began?

42 A parabola passes through the origin and the point (7, 88.2). An exponential function has an initial value of 40 and increases at a rate of 19.8% per year. If, for both functions, x is the number of years and $f(x)$ is the quantity in grams, which function will reach 200 grams first?

43 Given the following periodic function:
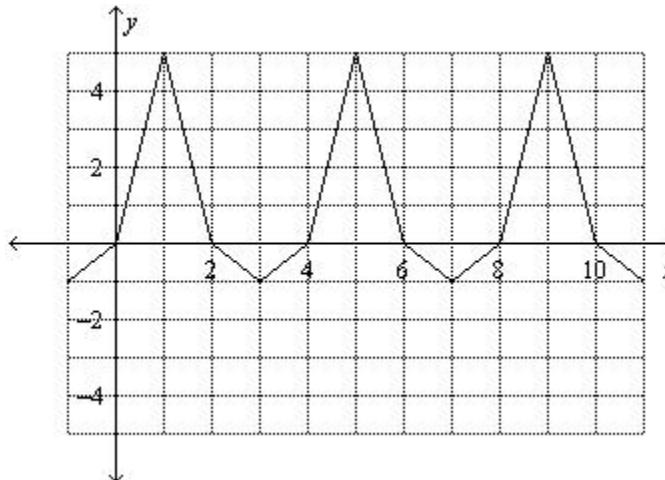
a) What is the period?
b) Determine $f(5)$
c) Determine $f(49.8)$

44

Given the following function: $f(x): \begin{cases} = 5x, \ 0 < x < 10 \\ = 4x + 9, \ 10 \le x < 100 \\ = 2.5x + 150, \ x \ge 100 \end{cases}$

Evaluate: $f(3) + f(9.9) + f(10) + f(25) + f(100) + f(240)$

45

Given: $f(x) = \dfrac{3}{8}x - 6$ Determine all of the properties.

46

To go to a rock concert with her friends, Joyce paid $214 for 4 balcony tickets and 3 main-floor tickets. To go to the same concert, Eric paid $208 for 5 balcony tickets and 2 main-floor tickets.

To go to this concert, how much will Kim pay for 2 main-floor tickets?

47

In a supermarket, a mix of 0.5 kg of peanuts and 0.75 kg of cashews costs $10.75. Likewise, a mix of 1 kg of peanuts and 1.25 kg of cashews costs $18.75.

According to this information, how much would a mix of 3.5 kg of peanuts and 2 kg of cashews cost?

48 An art supplies store sells three types of kits consisting of brushes and tubes of paint.

The "beginner's" kit contains 3 brushes and 5 tubes of paint and sells for $20.75.

The "professional's" kit contains 5 brushes and 10 tubes of paint and sells for $40.25.

The "expert's" kit contains 8 brushes and 20 tubes of paint.

Regardless of the kit, the price of one brush and the price of one tube of paint remains the same.

How much does the "expert's" kit cost?

49 John ordered slices of pizza and hamburgers for a group of 17 people. Each person chose only one item : either a slice of pizza for $1.20 or a hamburger for $1.80. John spent a total of $26.40.

How many slices of pizza and how many hamburgers did John order?

50 A car accelerates from a traffic light as soon as it turns green. The relationship between the time in seconds and the distance from the traffic light is a second degree polynomial function.

After 3 seconds the car was 72.9 metres from the stop light.

Another traffic light which is 300 metres from the first one turns green at the same time as the first traffic light and remains green for 6 seconds.

If the car continues to accelerate at the same rate, will it make it to the second light before it turns red?

Solutions to Review Questions

1	D
2	C
3	C
4	D
5	C
6	C

7   Option 3 because it depends the most on the number of sales made.

8

Monthly Internet Fees

Costs
($)

16 ——————————————————○———●

15 ——————————————○———●

13 ——————○———●

11 ——○———●

10 ●————————————————●

0   1   2   3   4   5   6   7   8   9   10   11

Number of hours on line
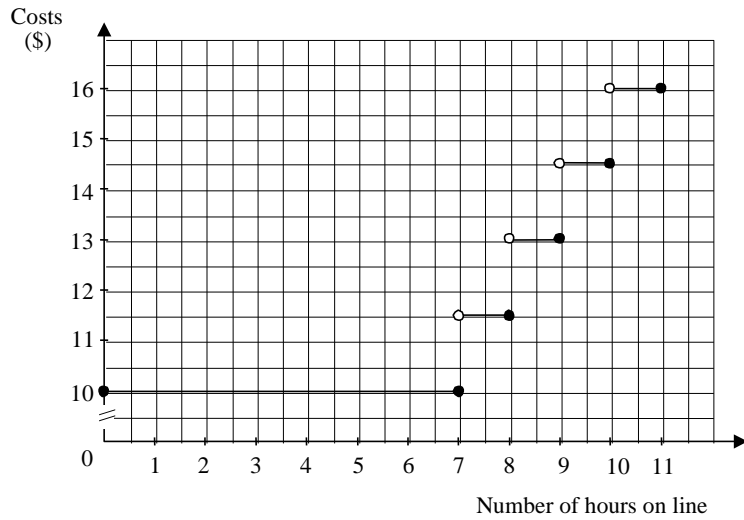
9

Money John Earns Washing
Windows at Customer's Home
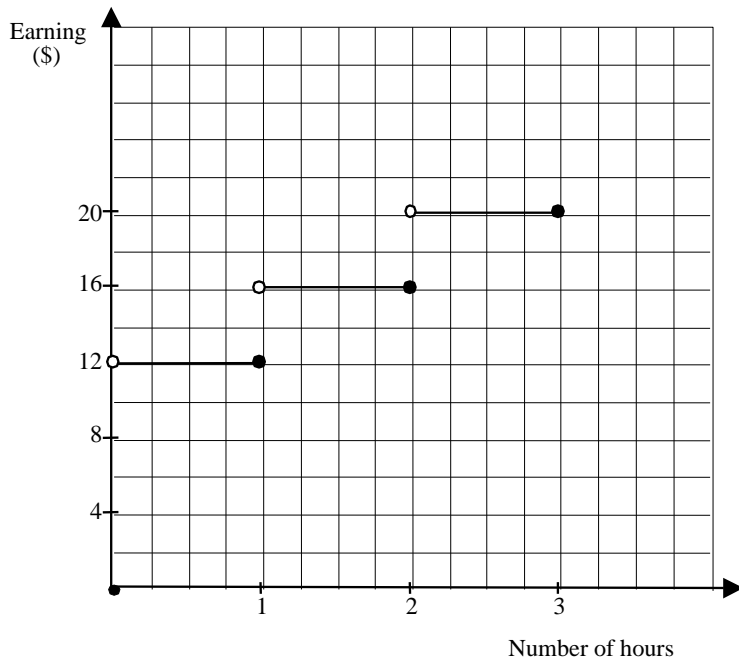
10

Mass of Block of Ice
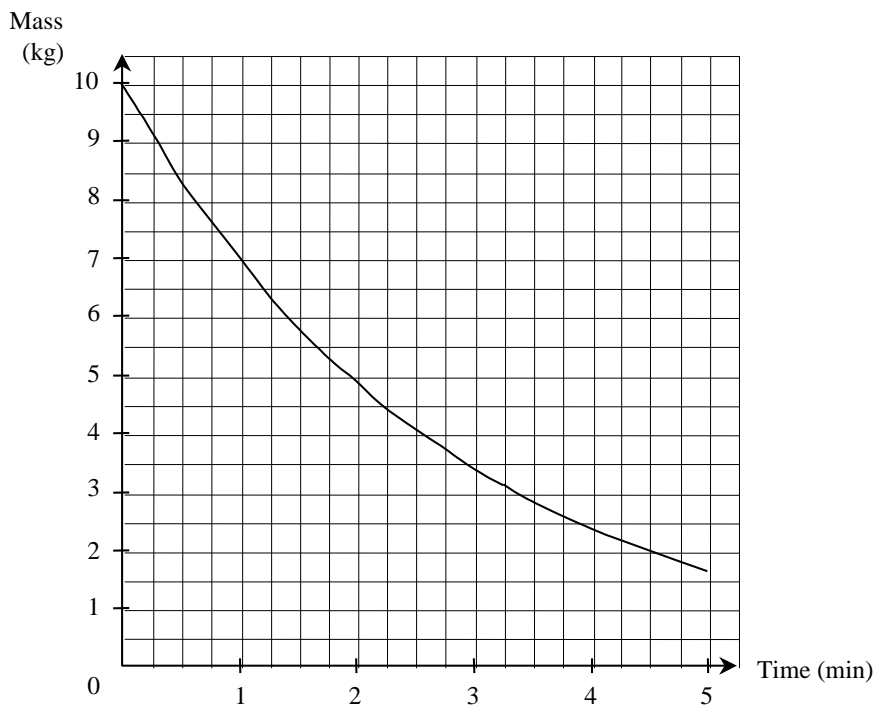
11 At the end of 2002, the value of the car was $13 784.20.

Cost
($)

**Cost of Ads in Relation to the
Number of Additional Words**

(25, 16)

Number of
additional words

---
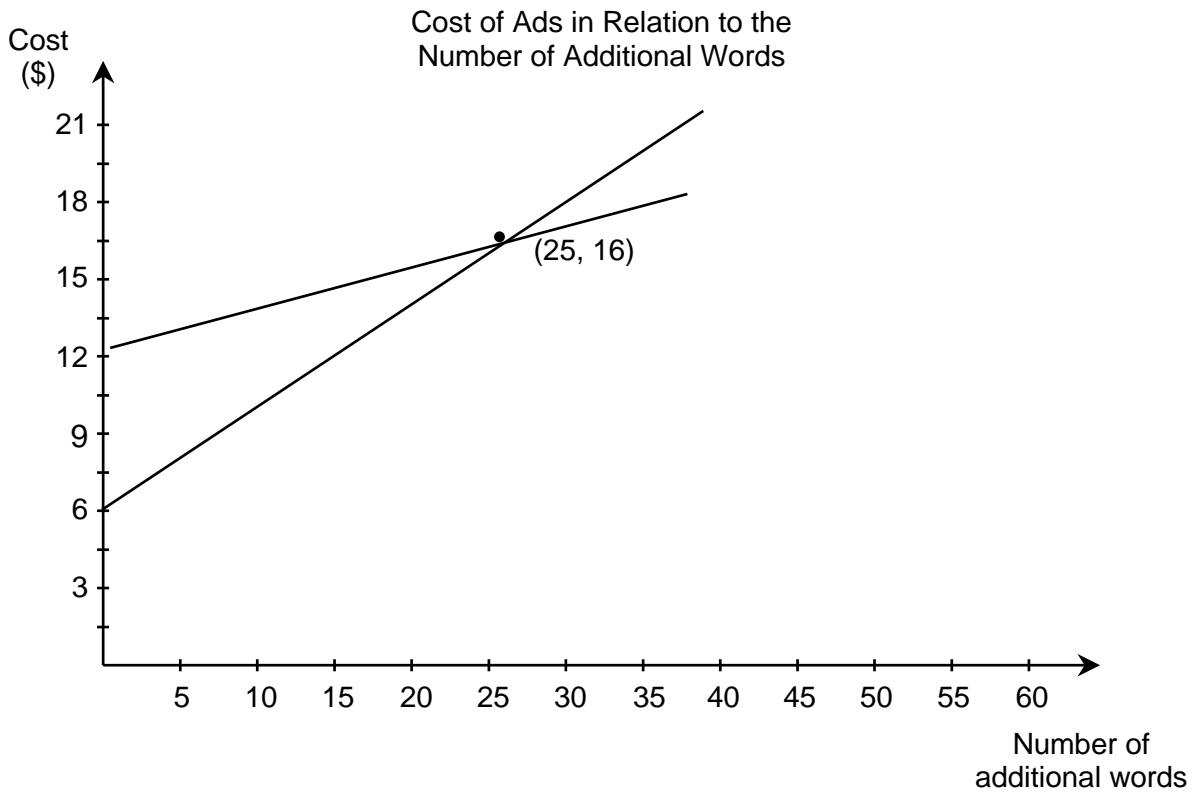
13 The total cost of the day tickets for this family of 4 will be $**95**.

---

14 At this rate, the speed of a processor on January 1, 2010, to the nearest MHz will be **4373** MHz.

---

15 a)

x	5	10	15	20	25
y	**500**	**250**	**167**	**125**	**100**

Note Do not penalize a student who wrote 166.6 for $x = 15$.

b) The type of relation is an **inverse relation**.

16

Gains with company A

Month	0	1	2	…	12
Sum ($)	100	105	110.25	…	179.59

After 1 year, the shares in company A are worth $179.59.

Gains with company B
        From the table of values, the shares of company B are worth $58.85.

Total gains
        $179.59 + 58.85 = 238.44$

Profit
        $238.44 - (100 + 75) = 63.44$

Answer: After 12 months, her profit is $63.44.

17   Example of an appropriate solution

Rental cost for the **1st** company
        Find the rate of change a:

$a = \dfrac{y_2 - y_1}{x_2 - x_1}$

$a = \dfrac{80 - 60}{200 - 0}$

$a = 0.1$

Rule
$y = 0.1x + 60$
Cost for 120 km
$y = 0.1 \times 120 + 60$
$y = 72$

Rental cost for the **2nd** company

Distance (km)	]80, 120]	]120, 160]
Cost ($)	60	80

Cost for **120** km: $**60**

The difference between the rental costs for a distance of 120 km
        $72 - $60 = $12

Answer   The difference in the rental costs for a distance of 120 km is $12.

18 Example of an appropriate solution

Country A	
Year	Number of people infected with AIDS
2005	1100
2006	1166
2007	1236
2008	1310
2009	1389
2010	1472

Country B	
Year	Number of people infected with AIDS
2005	800
2006	888
2007	986
2008	1094
2009	1215
2010	1348

Answer: In 5 years, there will be more people infected with AIDS in Country **A**.

19 Example of an appropriate method

• Distance George will drive

Cost under Plan B
      $164

Cost for the first 120 kilometres under Plan B
      120 km × $0.50/km = $60

Cost for additional kilometres
      $164 − $60 = $104

Number of additional kilometres
      $104 ÷ $0.40/km = 260 km

Number of kilometres George will drive
      120 km + 260 km = 380 km

Cost of rental under Plan A to travel 380 km

According to the graph, we can say that the rental cost will be $120 for a distance of 380 km.

Answer: If George chooses Plan A, he will have to pay $**120** to rent the car.

20 Example of an appropriate method

➤ Company A's rental charge for 4.5 hours

1st complete hour: $60 ⎤
2nd complete hour: $20 ⎥
3rd complete hour: $20 ⎬ total: $140
4th complete hour: $20 ⎥
Portion of 5th hour: $20 ⎦

➤ Company B's rental charge for 4.5 hours

Company A's rental charge for 4.5 hours + $25:

$140 + $25 = $165

➤ Company B's hourly rental charge after the 2nd hour

Charge for the first 2 hours: $130

Charge for the next 2.5 hours: $165 − $130 = $35

Hourly charge after the 2nd hour: $35 ÷ 2.5 hours = $14 per hour

Answer: After the 2nd hour, company B charges $**14** for each additional hour.

---

21 Examples of appropriate solutions
Stephan
Rule: $y = 1000(1.08)^x$

x (number of years)	y_1 (savings)
6	1586.9
7	1711.8
8	1850.9
9	1999
10	2158.9
11	2111.6
12	2518.2

Stephan will save $2000 in just over 9 years.
Emilie
Rate of savings: $120 per year; Initial value: $500
Rule
$$y = 500 + 120x$$
$$2000 = 500 + 120x$$
$$1500 = 120x$$
$$\frac{1500}{120} = 12.5 \text{ years}$$
Emilie will save $2000 in 12.5 years.
Answer: **Stephan** will be the first to save $2000.

22. Given the function $f(x) = \begin{cases} 3 & ,if\ x:[0,8[\\ -2x+4 & ,if\ x:[8,12[\\ -x^2 & ,if\ x:[12,20] \end{cases}$, determine which of the following false?

A) the function is positive over [0,8[B) the function is decreasing over [8,20]

C) the function has an initial value of 3   D) the function has a zero

23.

Plane 1: Period is 20
    $190 \div 20 = 9.5$,
    $0.5 \times 20 = 10$;
    So, $f(190) = f(10) = 200m$
Plane 2: Period is 40
    $190 \div 40 = 4.75$
    $0.75 \times 40 = 30$
    So, $f(190) = f(30) = 165m$
    $200 - 165\ m = \underline{35m\ apart}$

24.

$y = ax^2$ where y is the depth in meters and x is the time in seconds
$20 = a(2)^2$
$20 = 4a$
$a = 5$ $-> y = 5x^2$
               $y = 5(6)^2$
               $y = \underline{180m\ deep}$

25.
    a) What is the furthest a customer can travel if they are willing to pay up to $140 and how much would it cost?

        The furthest they can travel is 150km and it would cost $130

    b) Determine the cost of renting the car if a customer travels 260 km.

        step 4:]150, 200] $160; step 5:]200, 250] $190; step 6:]250, 300] $220
        It would <u>cost $220</u> to travel 260km (step 6)

26.

Week 1:	f(1200) = $300
Week 2:	f(2000) = 0.1(2000) + 300 = $500
Week 3:	f(4000) = 0.1(4000) +300 = $700
Week 4:	2100 – (300 + 500 + 700) = $600 *must be]1500, 4000]

$$600 = 0.1x + 300$$
$$300 = 0.1x$$
$$x = 3000$$

The employee had <u>$3000 in sales</u> in the fourth week.

27.

Exponential functions: $f(x) = a(base)^x$
Smallville: $f(x) = 25000(1.1)^x$ Bigtown: $41600 = 40000(b)^1$
$$b = 1.04$$

	$f(x) = 25000(1.1)^x$	$f(x) = 40000(1.04)^x$
Year #	Smallville population	Bigtown population
0 (2006)	25000	40000
1 (2007)	27500	41600
2 (2008)	30250	43264
3 (2009)	33275	44995
4 (2010)	36603	46794
5 (2011)	40263	48666
6 (2012)	44289	50613
7 (2013)	48718	
8 (2014)	53590	

<u>Bigtown reaches 50000 first</u>, in the sixth year, 2 years ahead of Smallville.

28

STATION 1	$1000 + 100x$
	$5000 - 1000 = 4000$
	$4000 \div 100 = 40$
$\Rightarrow$	40 segments of 30 seconds
STATION 2	$2000 + 50x$
	$5000 - 2000 = 3000$
	$3000 \div 50 = 60$
$\Rightarrow$	60 segments of 30 seconds
STATION 3	$150x$
	$5000 \div 150 = 33.3$
$\Rightarrow$	Can get 33 segments of 30 seconds

Final answer   The second radio station offers the best deal.

29 Let the initial price be $100.

	STORE 1	STORE 2
	10 % per day	10 % of the price announced the previous day
0	$100	$100
1	$90	$90 (10 % of 90)
2	$80	$81 (10 % of 81 = 8.10)
3	$70	$72.90 (10 % of 72.9 = 7.29)
4	$60	$65.61
	$60 (60 % of the initial price)	$65.61 (65.61 % of the initial price)

$$65.61 \% - 60 \% = 5.61 \%$$

Final answer : 5.61 %

30

Number of Hours Played	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Cost at Annie's Favourite Place $	0.00	3.00	6.00	9.00	12.00	15.00	18.00	21.00	24.00	27.00	30.00
Cost at Gaby's Favourite Place $	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00	15.00
Cost at Eric's Favourite Place $	0.00	6.00	6.00	12.00	12.00	18.00	18.00	24.00	24.00	30.00	30.00

Answer   Annie's favourite place offers the best deal because, in comparison to the others, a game is less expensive if it lasts a fraction of an hour.

31 The graph of the step function indicates that it will cost \$60 to travel between 200 and 250 km.
Rule of correspondence of the partial variation function

Let d: distance travelled in km
Let c: rental cost in \$

rate of change $= \dfrac{70-40}{200-0} = \$0.15/\text{km}$

fixed cost $= \$40 \qquad c = 0.15d + 40$

Rental cost for 240 km: $0.15 \times 240 + 40 = \$76$
Difference in cost: $\$76 - \$60 = \$16$
Answer   The exact difference in cost between the two plans is \$16.

32 Table of values

Number of days	M_1 (\$)	M_2 (\$)
0	0.01	1.00
1	0.03	2.00
2	0.09	4.00
3	0.27	8.00
4	0.81	16.00
5	2.43	32.00

The difference between the two amounts
$M_2 - M_1 = 32.00 - 2.43 = \$29.57$

Answer: The difference between the two amounts is \$29.57.

33 Amount charged by store A
$y_A = 4(1.15)^7 = \$10.64$
Amount charged by store B

Rate of change: $\dfrac{7.75 - 4.75}{5-1} = 0.75$

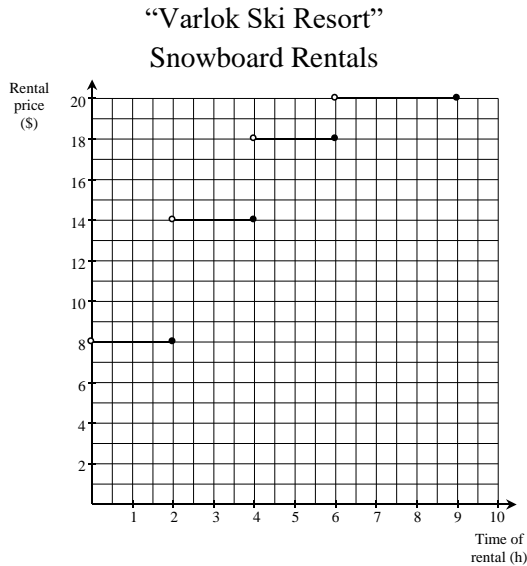| Number of days after the due date	Total amount charged
5	\$7.75
6	\$7.75 + \$0.75 = \$8.50
7	\$8.50 + \$0.75 = \$9.25

Difference between the two amounts
$10.64 - 9.25 = \$1.39$

Answer: The difference between the total amounts charged by these stores for a movie returned 7 days after the due date is \$1.39.

34

Time of rental (hours)	Total cost ($)
]0, 2]	8
]2, 4]	14
]4, 6]	18
]6, 9]	20

"Varlok Ski Resort"
Snowboard Rentals

Five criteria to consider:
1. The overall graph is correct.
2. The axes are identified correctly.
3. The axes are scaled correctly.
4. The last segment ends after 9 hours.
5. The segments are open on the left.

35   Gains with company A

Month	0	1	2	…	12
Sum ($)	100	105	110.25	…	179.59

After 1 year, the shares in company A are worth $179.59.

Gains with company B

From the table of values, the shares of company B are worth $58.85.

Total gains

$179.59 + 58.85 = 238.44$

Profit

$238.44 - (100 + 75) = 63.44$

Answer: After 12 months, her profit is $63.44.

36 • Rule representing the cost of a delivery carried out by Company B
Rate of change

$$\frac{\$3.20 - \$8.00}{8\,km - 20\,km} = \frac{-\$4.80}{-12\,km}$$
$$= \$0.40/km$$

Since the delivery cost is proportional to the distance travelled

$$y_B = 0.40x$$

x: distance travelled to deliver a parcel, in km
y_B: cost of delivery carried out by Company B, in $

• Distance for which the delivery cost is the same for both companies

We are looking for the value of x for which $y_A = y_B$.

x	$y_A = 0.10x + 4.50$	$y_B = 0.40x$
0	4.5	0
20	6.5	8
18	6.3	7.2
15	6	6

The delivery cost for a distance of 15 km is the same for both companies.

Answer: The delivery cost for a distance of **15** km is the same for both companies.

37 • Amount of water in the two basements 20 hours after the pumping began

$$y_S = 15\,000 - 300x$$

x: time elapsed from the moment the pumping began, in hours
y_S: amount of water in Smith's basement, in litres

After 20 hours spent pumping

$$y_S = 15\,000 - 300(20)$$
$$= 9000$$

After 20 hours spent pumping, the two basements each contained 9000 litres of water.
• Number of litres of water in Black's basement before the pumping began
20 hours after the pumping began, there were still 9000 litres of water
In 20 hours, the pump remove 20×200 litres $= 4000$ litres of water
Number of litres of water before the pumping began

$$9000\,L + 4000\,L = 13\,000\,L$$

Answer: There were **13 000** litres of water in Black's basement before the pumping began.

38

x : number of km driven

y : cost to rent in dollars

Rate of variation (cost per km)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{544 - 379}{900 - 600}$$

$$= 0.55$$

Equation representing rental cost

$$y = .55x + b$$
$$544 = .55(900) + b$$
$$544 = 495 + b$$

Fixed cost $= \$49$

$$\therefore y = .55x + 49$$
$$y = .55(1300) + 49$$
$$y = 715 + 49$$
$$y = \$764$$

Answer: It would cost Carlo $\$\mathbf{764}$ to rent a car for a 1300-km trip.

39

Determine the linear relation defining the cost of printing photos at PICS Я US $y = ax + b$

Cost per photo

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$

$$a = \frac{19.25 - 10.25}{35 - 15}$$

$$a = \frac{9}{20}$$

$$a = 0.45$$

Initial value

$$y = ax + b$$
$$10.25 = 0.45(15) + b$$
$$b = 3.50$$
$$y = 0.45x + 3.50$$

Determine the cost of printing 28 photos at PICS Я US

$$= 0.45(28) + 3.50$$
$$= \$16.10$$

Determine the cost of printing 28 photos at FOTOFLASH

$$= 0.62 \times 28$$
$$= 17.36$$

Determine the difference between the two shops

$$\$17.36 - \$16.10 = \$1.26$$

Answer: Samantha will save $\$\mathbf{1.26}$ by having her photos developed at the store that gives her the better deal.

40	x (number of years)	y (remaining mass)
	0	1000
	1	800
	2	640
	3	512
	4	409.6

Answer: In 4 years, the mass of the radioactive substance will be **409.6** kg.

41   Rate of change for Lake Hershey

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{2200 - 1600}{8 - 5}$$

$$= \frac{600}{3}$$

$$= 200$$

Initial value

$$y = ax + b$$
$$y = 200x + b$$
$$1600 = 200(5) + b$$
$$1600 - 1000 = b$$
$$600 = b$$
$$y = 200x + 600$$

Lake Morin

$$y = 16x^2$$
$$= 16(14)^2$$
$$= \mathbf{3136}$$

Lake Hershey

$$200(14) + 600 = \mathbf{3400}$$

Answer: **Lake Hershey** was more polluted, 14 years after the study began.

	Parabola y=1.8x^2	Exponential y=40(1.198)^x
0	0	40
1	1.8	47.92
2	7.2	57.40816
3	16.2	68.77497568
4	28.8	82.39242086
5	45	98.7061202
6	64.8	118.249932
7	88.2	141.6634185
8	115.2	169.7127754
9	145.8	203.3159049
10	180	243.5724541
11	217.8	291.7998

The exponential function will reach 200 grams first.

a)   Period = 4

b) $f(5) = 5$

c) $f(49.8) = f(1.8)$ equation of line from $x = 1$ to $x = 2$ is $y = -5x + 10$

so, $f(1.8) = -5(1.8) + 10$

$f(1.8) = 1$

$f(3) = 15$

$f(9.9) = 49.5$

$f(10) = 49$

$f(25) = 109$

$f(100) = 400$

$f(240) = 750$ **Total: 1372.5**

Domain & Range: all real numbers, No maximum or minimum
Increasing over all real numbers
Zero: $x = 16$ **Negative**:]-∞, 16]   **Positive**: [16, ∞[

46 Given x : price of one balcony ticket

y : price of one main-floor ticket

System of equations representing this situation

$4x + 3y = 214$

$5x + 2y = 208$

Solution of the system of equations

$x = 28$ $y = 34$

Price of Kim's tickets   Result : Kim will pay $68 for 2 main-floor tickets.

$2 \times 34 = 68$

47 System of equations

x : price of 1 kg of peanuts

y : price of 1 kg of cashews

$0.5x + 0.75y = 10.75$

$x + 1.25y = 18.75$

By solving the system, $x = 5$ and $y = 11$.

Therefore, 1 kg of peanuts costs $5 and 1 kg of cashews costs $11.

Price of the new mix

$(3.5 \times 5) + (2 \times 11) = \$39.50$

Result : The price of the new mix is $39.50.

48 Let x : the price of one brush

y : the price of one tube of paint

System of equations

$3x + 5y = 20.75$

$5x + 10y = 40.25$

Solution of the system of equations

$x = 1.25$ $y = 3.40$

Price of the "expert's" kit   Result : The "expert's" kit costs $78.

$8 \times 1.25 + 20 \times 3.40 = 78$

49 Let x : the number of slices of pizza

y : the number of hamburgers

System of equations

$x + y = 17$

$1.20x + 1.80y = 26.40$

Solution of the system of equations

$x = 7$ $y = 10$ Result : John ordered 7 slices of pizza and 10 hamburgers.

50 Rule: $y = 8.1x^2$

In 6 seconds: $y = 8.1(6)^2$

$y = 291.6$ metres   It will not make it to the green light on time.