Name : ___________________________________

Math 4 SN – Midyear Review (December)

1 A swimming pool containing 30 000 litres of water is emptied by means of a pump which pumps water at a constant rate.

The pump is turned on at 8:00 a.m. By noon there are still 22 000 litres of water in the pool.

At this rate, at what time will the pool be completely empty?

2 A manufacturer sells beach umbrellas for \$150 each. Each one costs \$70 to make. He calculates that all other fixed costs (rent, salaries,...) come to \$16 000 per month.

How many beach umbrellas must he sell per month to earn a profit of \$1200?

3 Water is dripping into a barrel from the broken faucet of a water tank.

At certain times of the day, the total amount of water collected in the barrel is noted and recorded in the following table.

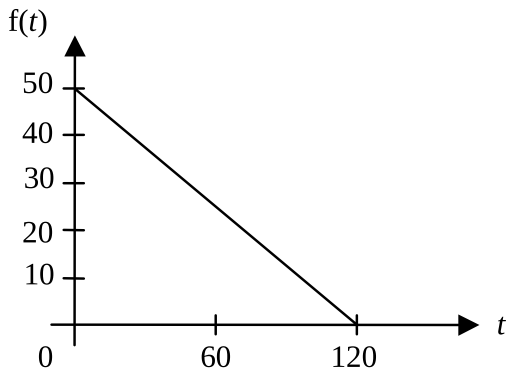
Time of observation	08:00	09:30	10:00	12:00	14:30
Amount of water in the barrel (in litres)	7	11.5	13	19	26.5

At 14:30, 27 litres of water remain in the tank.

In how many hours will the tank be empty?

4 A bathtub containing 50 L of water empties in 2 minutes. The graph to the right represents the number of litres of water remaining in the bathtub after *t* seconds.

How many litres of water remain in the bathtub after 48 seconds?



5 The grad committee spent a certain amount of money to rent a hall for their grad dance. If 200 students attend, the profit will be \$200. If 300 students attend, the profit is \$700.

What equation can be used to calculate the profit *p* as a function of the number of students *s* who attend the dance?

6 A weather office recorded the snow accumulation during a storm. By 8 a.m., there was 180 cm of snow on the ground; by 12 noon, there were 190 cm of snow on the ground.

If the storm began at midnight (0 hours) the same day and the snow fell at a steady rate for the whole storm, how much snow was on the ground by 6 a.m.?

7 A school organizes a lottery to enable it to buy a computer. If each ticket sells for $2, the school will be short $1000. However,if they sell each ticket for $3, the school will make a profit of $250. How many tickets does the school need to print and what is the price of the computer?

8 At an amusement park, there is an entry fee and a charge for each ride on the merry-go-round.

Three friends went to the park. Susan spent a total of $10.50 for the entry and 10 trips on the merry-go-round. Maria spent $13.50 to enter and take 14 rides on the merry-go-round.

How much did Sophie have to pay for the entry fee and 8 rides on the merry-go-round?

9 Two partners each invested a certain sum of money to open a shop. The amount invested by the first was double the amount invested by the second. If the first partner had tripled his investment and the second had doubled his, they would have had $130 000 available to open the shop.

What was the sum total invested by the two partners?

10 Edith bought some small tiles and some large tiles to recover a floor. She laid a total of 272 tiles. The design required 2.4 times as many large tiles as small ones.

At the end of the job, 40 small tiles and 16 large tiles were left over.
How many large tiles had she brought?

11 The length of a rectangle is 8 cm more than its width. Its perimeter is 240 cm.

What is the area of this rectangle?

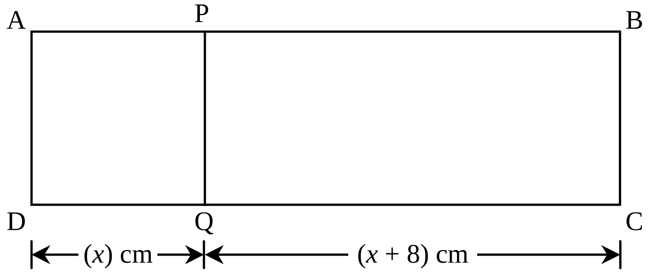
12 Raymond rents cabins during ice fishing season. In the winter of 1997, when it cost $75 to rent a cabin, he received an average of 50 customers a day. In 1998, Raymond will lower the rent by x dollars and expects that the number of customers will increase by twice the value of x.

Let R be the quadratic function that represents the daily cabin rental income.

What is the maximum daily income that Raymond can earn in 1998?

13 In the figure below, line segment PQ divides rectangle ABCD into the following two quadrilaterals: square APQD and rectangle PBCQ.

The area of rectangle ABCD is 120 cm^2. In addition, m $\overline{DQ}$ = (x) cm and m $\overline{QC}$ = (x + 8) cm.



What is the numerical area of rectangle PBCQ?

14 Functions f and g are represented by parabolas in the Cartesian plane below.

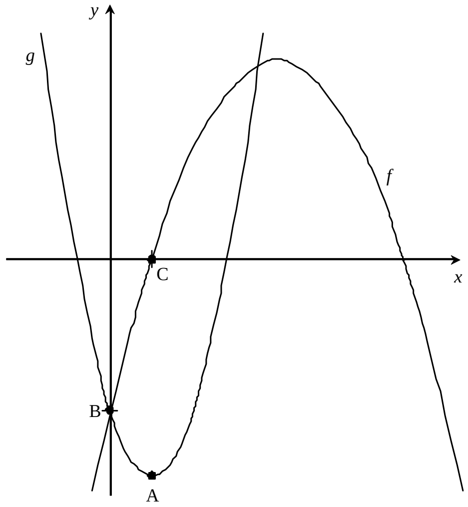
The parabola that represents function f passes through points B and C. The parabola that represents function g passes through points B and A. Point A is the vertex of the parabola that represents function g.

Point B is located on the y-axis. Point C is located on the x-axis.

The rule of function f is $f(x) = -0.25x^2 + 4x - 7$.

The x-coordinate of point A is the same as the x-coordinate of point C.

The minimum of function g is -10.
What is the rule of function g?



15 Annie and Mark each borrowed some money interest free. Annie borrowed \$500 which she will repay at a rate of \$40 per month. Mark borrowed \$600 which he will repay at a rate of \$60 per month. They both make their first payment at the same time.

After how many months will Annie's debt be equal to Mark's?

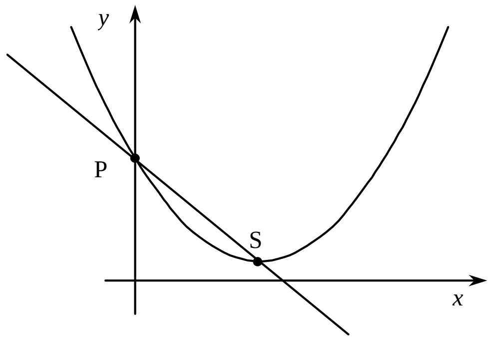
16 A group of students has to sell a certain number of sweaters in order to raise money for an extra-curricular activity. If each student were to take five sweaters to sell, there would be 20 sweaters left over. However if each student were to take 6 sweaters there would be 11 sweaters missing.

How many students are in the group and how many sweaters are for sale?

17 There are 120 tennis balls in a bag : some are white, some are green and some are yellow. The number of white balls equals the number of yellow balls. If there were 20 more green balls, the number of green balls would be double the number of white balls.

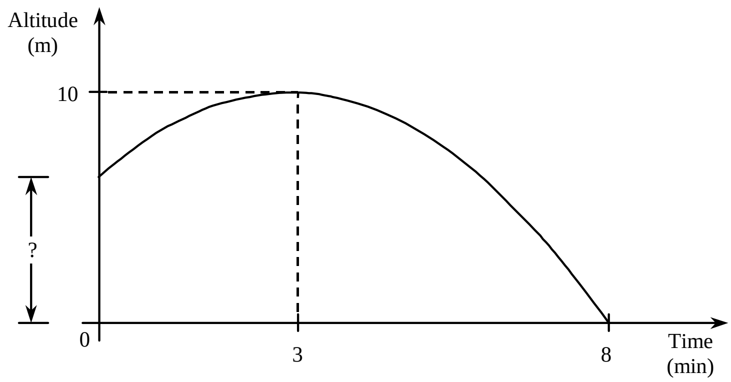
How many balls of each colour are there in the bag?

18 The equation of the parabola shown below is $y = 4x^2 - 40x + 101$.
Point S is the vertex of this parabola. The parabola intersects the y-axis at point P.



What is the equation of the line passing through points P and S?

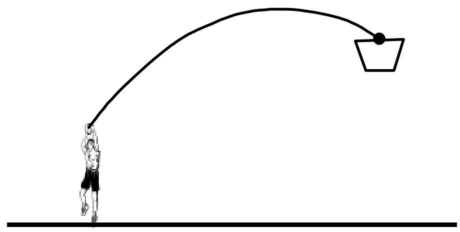
19 Melanie was playing with a remote-controlled toy airplane. The plane took off from a balcony and landed on the ground 8 minutes later. Three minutes after taking off, the plane reached a maximum altitude of 10 metres. In the graph below, the plane's altitude as a function of time is represented by a portion of a parabola.



How high off the ground is the balcony located?

20 Caroline throws a ball toward a basket located 3 m above the ground.

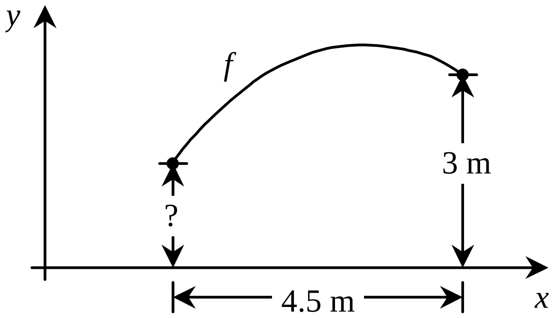
The ball reaches a maximum height. On its way down, it enters the basket.



In the Cartesian plane on the right, the side view of the ball's trajectory is represented by function f. The scale of this graph is in metres.

The rule associated with function f is $f(x) = -0.2(x - 5)^2 + 3.45$.

The horizontal distance between Caroline and the location of the basket is 4.5 m.



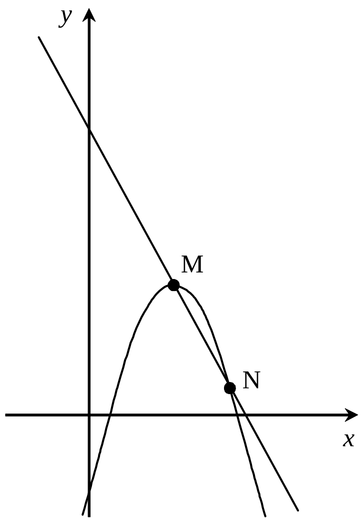
At the moment that Caroline throws the ball, what is the distance between the ball and the ground?

21 In the Cartesian plane on the right, a straight line and a parabola intersect at points M and N.

The equation of the parabola is $y = -2x^2 + 12x - 8$.

Point M is the vertex of the parabola.

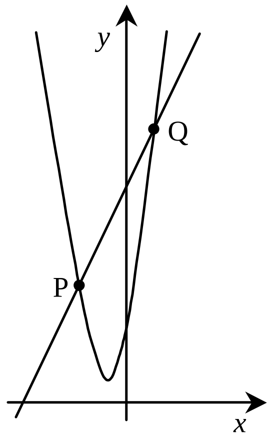
The y-intercept of the line is 22.



What are the coordinates of point N?

22 Points P and Q are the points of intersection of the line and the parabola drawn in the Cartesian plane on the right.

The equation of the line is $y = 2x + 21$.



The following table of values shows the coordinates of different points located on the parabola above.

x	y
-4	6
-3	3
-2	2
-1	3
0	6

What are the coordinates of the points of intersection P and Q?

23 The value v, in dollars, of a stock varies with time t, in days. This situation is defined by

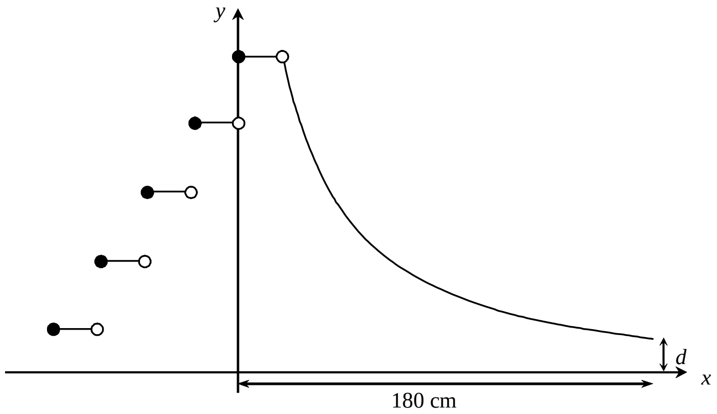
$$v(t) = -\left[\frac{t}{2}\right] + 8 \quad \text{if} \quad t \in [0, 6], ,$$

$$v(t) = \frac{3}{16}(t - 10)^2 + 2 \quad \text{if} \quad t \in [6, 10], ,$$

$$v(t) = 0.25t - 0.5 \quad \text{if} \quad t \in [10, 20],$$

For how many days was the value v of the stock \$3.92 or less?

24 A designer is preparing a model of a children's slide. She began by drawing the steps and the slide on a Cartesian plane scaled in cm, as shown in the diagram below.



The steps of the slide are represented by the relation $y = 32.5[0.05x + 3] + 52.5$.
The top step begins on the y-axis. The slide is attached to the other end of the top step.

The slide is represented by a quadratic function with the equation $y = a(x - 200)^2 + 30$.
The end of the slide is 180 cm from the origin of the Cartesian plane.
To the nearest tenth of a centimetre, what is the distance (d) from the ground to the end of the slide?

25 The function f is defined by the following rule:

$$f(x) = 3\left[-\frac{(x-1)}{2}\right] + 6$$

What are the zeros of this function?

A)]1.5, 2[

B)]3, 5]

C)]5, 8[

D) [5, 8[

26 The *Mount Tessa Ski Resort* bases its prices on the time skiers spend on the hill, according to the following equation:

$$c(t) = 1.50\left[\frac{t+30}{20}\right]$$

where t represents the time, in minutes, on the hill
and c represents the cost of the ticket, in dollars

If James is on the hill for 2 hours, how much will he pay for his ticket?

A) $5.25

B) $10.50

C) $11.25

D) $12.00

27 The graph of a function is shown below.

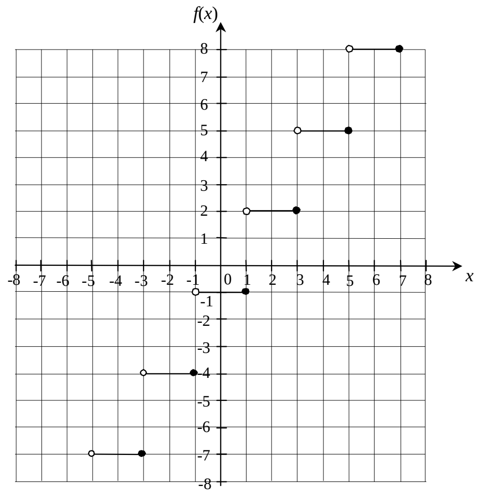
Which of the following equations describes the function shown in the graph?

A) $f(x) = -3[2(x-1)] - 1$

B) $f(x) = -3[-2(x-1)] - 1$

C) $f(x) = -\frac{3}{2}[x-1] - 1$

D) $f(x) = -3\left[-\frac{1}{2}(x-1)\right] - 1$



28 Rhys, Kim and Lucas are working on their math homework, but each one of them has come up with a different answer when simplifying the expression below.

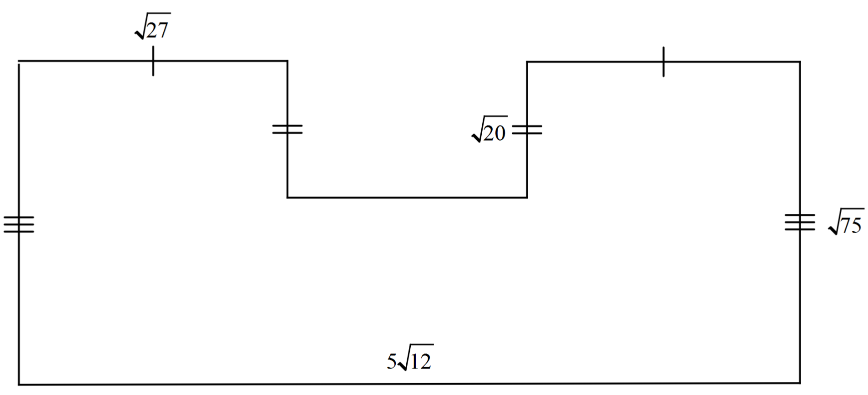
$$\frac{5x^2}{3} \div \left(\frac{x^2+2x-3}{x^2-1} + \frac{2x+6}{3x+3} \right)$$

Rhys' answer was $\dfrac{x+3}{x^3+1}$ Lucas' answer was $\dfrac{x^3+x^2}{x+3}$

Kim's answer was $\dfrac{5x^3+5x^2}{9x+27}$

Which one of them, if any, had the correct answer? Show all of your work.

29 Express the perimeter of the following figure in simplified radical form:



Math 4 SN – Midyear Review Answer Key

1 Pump rate

$$\frac{30\ 000\text{ L} - 22\ 00\text{ L}}{12\text{ h} - 8\text{ h}} = 2000\text{ L/h}$$

Time required to empty the 22 000 litres remaining

$$\frac{22\ 000\text{ L}}{2000\text{ L/h}} = 11\text{ h}$$

Time when the pool will be empty.

12:00 + 11:00 = 23:00

2 Given n the number of umbrellas
$P(n)$ the profit

Equation: $P(n) = (150 - 70)n - 16\ 400$

Solution of the equation

$$1200 = 80n - 16\ 400$$

$$n = n = 220$$

Result The manufacturer must sell <u>220</u> beach umbrellas to make a profit of $1200.

3 Amount of water that drips out each hour (rate of change)

$$\frac{11.5 - 7}{9.5 - 8} = 3$$

Number of hours needed to empty the tank

$27 \div 3 = 9$

Result The tank will be empty in 9 hours.

4 Calculation of the slope

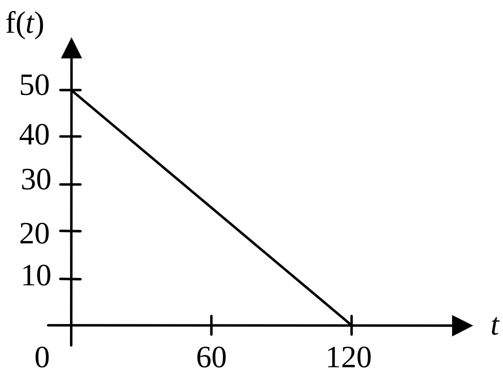
$$m = \frac{0 - 50}{120 - 0}$$

$$= \frac{-5}{12}$$

Equation of the line

$$f(t) = mt + b$$

$$= \frac{-5}{12}t + 50$$



Calculation of f(t) when $t = 48$

$$f(t) = \frac{-5}{12}t + 50$$

$$f(48) = \frac{-5}{12}(48) + 50$$

$$= 30$$

Result After 48 seconds, 30 L of water remain in the bathtub.

$\boxed{5}$ Calculation of the rate of change

Using the ordered pairs (200, 200) and (300, 700), we obtain

$$\frac{700 - 200}{300 - 200}$$

$$= \frac{500}{100}$$

= \$5 per student (A student who omitted to write the unit of measure should not be penalized.)

Calculation of b

Substituting the ordered pair (200, 200) in p(s) = 5s + b, we obtain

$$\begin{aligned} 200 &= 5 \times 200 + b \\ 200 &= 1000 + b \\ 200 - 1000 &= b \\ -800 &= b \end{aligned}$$

Substituting - 800 in p(s) = 5s + b, we obtain

$$p(s) = 5s - 800$$

Result The equation is p(s) = 5s – 800.

$\boxed{6}$ Given x : time of the day

$f(x)$: snow accumulation

Using the ordered pairs (8, 180) and (12, 190), we obtain

$$\frac{190 - 180}{12 - 8}$$

$$= \frac{10}{4}$$

= 2.5 cm per h

Calculation of b

Substituting the ordered pair (8, 180) in f(x) = 2.5x + b, we obtain

$$\begin{aligned} 180 &= 2.5 \times 8 + b \\ 180 &= 20 + b \\ 180 - 20 &= b \\ 160 &= b \end{aligned}$$

Calculating f(6) in f(x) = 2.5x + 160, we obtain 175.

Result By 6:00 a.m., there were 175 cm of snow on the ground.

$\boxed{7}$ Given x, the number of tickets

y, the price of the computer

System of equations

$$2x = y - 1\,000$$

$$3x = y + 250$$

Solution of the system of equations

x = 1250 and y = 3500

Result : The school has to print <u>1250</u> tickets and the computer costs \$<u>3500</u>.

$\boxed{8}$ System of equations x : entry fee

y : cost of a ride on the merry-go-round

$x + 10y = 10.50$

$x + 14y = 13.50$

Solution of the system of equations

$x = \$3.00$

$y = \$0.75$

Determining the cost of entry and 8 rides on the merry-goround

$C = x + 8y = 3.00 + 8 \times 0.75 = 9.00$

$C = 9.00$

Result : $9.00

$\boxed{9}$

$f = 2s$

$3f + 2s = 130\ 000$

Sum invested by each partner

$f - 2s = 0$

$3f + 2s = 130\ 000$

$4f = 130\ 000$

$f = 32\ 500$

$2s = 32\ 500$

$s = 16\ 250$

Sum total invested by the two partners

$32\ 500 + 16\ 250 = 48\ 750$

Result : The sum total invested by the two partners was $48 750.

$\boxed{10}$ Given x, the number of large tiles used

y, the number of small tiles used

The system of equations :

$x + y = 272$

$x = 2.4y$

Solution of the system of equations $x = 192$ and $y = 80$

The total number of large tiles bought

$192 + 16 = 208$

Result : She had bought 208 large tiles.

$\boxed{11}$ Given x : length of the rectangle

y : width of the rectangle

System of equations representing the situation

$x - y = 8$

$2x + 2y = 240$

Solution of the system of equations $x = 64$ $y = 56$

Area of the rectangle

$64 \times 56 = 3\ 584$

Result : The area of the rectangle is 3584 cm^2.

12 Let $75 - x$: the cost of renting a cabin in 1998
Let $50 + 2x$: daily number of customers in 1998

$R = (75 - x)(50 + 2x)$
$R = -2x^2 + 100x + 3750$

A parabola that opens downward ($a < 0$) represents the set of all points in the Cartesian plane that are defined by the above relation. The y-coordinate of the vertex corresponds to maximum daily income that Raymond can earn in 1998.

y - coordinate of the vertex: 5000

Answer The maximum daily income is \$5000.

13 **Polynomial representing the area of rectangle ABCD**
Since APQD is a square, segment AD measures (x) cm.
Area of rectangle ABCD

$$\mathrm{m}\ \overline{\mathrm{AD}} \times \mathrm{m}\ \overline{\mathrm{DC}}$$
$$\mathrm{m}\ \overline{\mathrm{AD}} \times \left(\mathrm{m}\ \overline{\mathrm{DQ}} + \mathrm{m}\ \overline{\mathrm{QC}}\right)$$
$$x(x + x + 8)$$
$$x(2x + 8)$$
$$2x^2 + 8x$$

Value of x
Area of rectangle ABCD = 120 cm^2

$$\begin{aligned}
2x^2 + 8x &= 120 \\
2x^2 + 8x - 120 &= 0 \\
2(x^2 + 4x - 60) &= 0 \\
x^2 + 4x - 60 &= 0 \\
x^2 - 6x + 10x - 60 &= 0 \\
x(x - 6) + 10(x - 6) &= 0 \\
(x - 6)(x + 10) &= 0 \\
x &= 6 \quad \text{or} \quad x = -10 \text{ (impossible)}
\end{aligned}$$

Area of rectangle PBCQ

$$\mathrm{m}\ \overline{\mathrm{PQ}} \times \mathrm{m}\ \overline{\mathrm{QC}}$$
$$x(x + 8)$$
$$6(6 + 8)$$
$$84 \text{ cm}^2$$

Answer: The numerical area of rectangle PBCQ is **84** cm^2.

14 x-coordinate of point C

$0 = -0.25x^2 + 4x - 7$
$0 = -0.25(x^2 - 16x + 28)$
$0 = (x - 2)(x - 14)$
$x = 2 \quad \text{or} \quad x = 14$

The x-coordinate of point C is 2.

Coordinates of point A

The x-coordinate of point A is the same as the x-coordinate of point C (i.e. 2).
The y-coordinate of point A is -10.

Coordinates of point A
A(2, -10)

Coordinates of point B

x-coordinate of point B: 0
y-coordinate of point B: $f(0) = -7$

Coordinates of point B
B(0, -7)

Rule of function g

$$g(x) = a(x - 2)^2 - 10$$
$$-7 = a(0 - 2)^2 - 10$$
$$-7 = 4a - 10$$
$$\frac{3}{4} = a$$
$$g(x) = \frac{3}{4}(x - 2)^2 - 10$$

Answer: The rule of function g is $g(x) = \frac{3}{4}(x - 2)^2 - 10$.

15 Rule for calculating Annie's debt $D_1(x)$ as a function of the number of months elapsed x

$D_1(x) = 500 - 40x$

Rule for calculating Mark's debt $D_2(x)$ as a function of the number of months elapsed x

$D_2(x) = 600 - 60x$

Number of months elapsed when $D_1(x) = D_2(x)$

$500 - 40x = 600 - 60x$
$20x = 100$
$x = 5$

Result : After 5 months, Annie's debt will be equal to Mark's.

16 Given n, the number of students s, the number of sweaters

System of equations is

$5n + 20 = s$

$6n - 11 = s$

Solution of the system of equations

$5n + 20 = 6n - 11$

$n = 31$

$s = 175$

Result : The number of students in the group is 31.
The number of sweaters is 175.

17 Let x : represent the number of white balls y : represent the number of green balls
x : represent the number of yellow balls

The system of equations

$2x + y = 120$

$y + 20 = 2x$

Solution of the system of equations

$2x + y = 120$

$2x - y = 20$

$4x = 140$

$x = 35$ and $y = 50$

Result : The number of white and yellow balls is 35 each and the number of green balls is 50.

18 **Coordinates of point P**

If $x = 0$ then $y = 4(0)^2 - 40(0) + 101 = 101$ P(0, 101)

Coordinates of point S

The x-coordinate of the vertex of the parabola: $= 5$

The y-coordinate of the vertex of the parabola: $= 1$ S(5, 1)

Slope of the line passing through P and S

slope: $\dfrac{101 - 1}{0 - 5} = -20$

***y*-intercept of the line passing through P and S**

The y-intercept of the line is the same as that of the parabola (i.e. 101).

Answer The equation of the line passing through points P and S is $y = -20x + 101$.

19 **Rule of the function** x: time in minutes $f(x)$ = altitude in metres

$f(x) = a(x - h)^2 + k$

$f(x) = a(x - 3)^2 + 10$

$f(8) = 0$ then $0 = a(8 - 3)^2 + 10$

$0 = a(25) + 10$

$\dfrac{-10}{25} = a$

$-0.4 = a$

$f(x) = -0.4(x - 3)^2 + 10$

***y*-intercept** $f(0) = -0.4(0 - 3)^2 + 10 = 6.4$

Answer The balcony is located 6.4 m off the ground.

20 ***x*-coordinate of the location of the basket**

y-coordinate of the location of the basket: 3

$$-0.2(x-5)^2 + 3.45 = 3$$
$$-0.2(x-5)^2 = -0.45$$
$$(x-5)^2 = 2.25$$
$$x - 5 = -1.5 \quad \text{or} \quad x - 5 = 1.5$$
$$x = 3.5 \qquad\qquad x = 6.5$$

Since the basket is located to the right of the vertex of the parabola, $x = 6.5$.
x-coordinate of the location of the basket: 6.5

***y*-coordinate of the location of the ball at the moment Caroline throws it**

x-coordinate of the location of the ball at the moment Caroline throws it: $6.5 - 4.5 = 2$

$$f(2) = -0.2(2-5)^2 + 3.45 = 1.65$$

y-coordinate of the location of the ball at the moment Caroline throws it: 1.65

Answer: At the moment that Caroline throws the ball, the distance between the ball and the ground is **1.65** m.

21 Coordinates of point M: M(3, 10)

Equation of line MN
Slope

$$\frac{22-10}{0-3} = \frac{12}{-3}$$
$$= -4$$

y-intercept : 22 Equation of line MN $y = -4x + 22$

Coordinates of point N

$$\left.\begin{array}{l} y = -2x^2 + 12x - 8 \\ y = -4x + 22 \end{array}\right\} \Rightarrow \begin{array}{r} -2x^2 + 12x - 8 = -4x + 22 \\ -2x^2 + 16x - 30 = 0 \\ -2(x^2 - 8x + 15) = 0 \\ (x-3)(x-5) = 0 \\ x = 3 \quad \text{or} \quad x = 5 \end{array}$$

If $x = 3$, then $y = -4(3) + 22 = 10$. This would be point M.
If $x = 5$, then $y = -4(5) + 22 = 2$. This would be point N.

Coordinates of point N: N(5, 2)

Answer: The coordinates of point N are N**(5, 2)**.

22 The equation of the parabola is in the form $y = a(x - h)^2 + k$. Given the symmetry observed in the table of values, h = -2 and k = 2.

$y = a(x + 2)^2 + 2$
$6 = a(0 + 2)^2 + 2$
$4 = 4a$
$1 = a$
$y = (x + 2)^2 + 2$ or $y = x^2 + 4x + 6$

Points of intersection of the line and the parabola

$x^2 + 4x + 6 = 2x + 21$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$x = -5$ or $x = 3$
If $x = -5$ then $y = 2(-5) + 21 = 11$
If $x = 3$ then $y = 2(3) + 21 = 27$

Answer: The coordinates of the points of intersection are P(**-5, 11**) and Q(**3, 27**).

23 Answer: 10.88 days

24 Greatest integer function

$$x = 0 \Rightarrow \quad y = 32.5[0.05(0) + 3] + 52.5$$
$$= 32.5[3] + 52.5$$
$$= 150 \text{ cm}$$

Step length = $\frac{1}{0.05} = 20 \Rightarrow$ Last open point is (20, 150)

Quadratic function

$$150 = a(20 - 200)^2 + 30$$
$$120 = 32400a \qquad \text{Equation} \qquad y = \frac{1}{270}(x - 200)^2 + 30$$
$$\frac{1}{270} = a$$

$$x = 180 \Rightarrow \quad y = \frac{1}{270}(180 - 200)^2 + 30$$
$$y = 31.48$$

Answer: To the nearest tenth of a centimetre, the distance is **31.5** cm.

25 B

26 B

27 D

28 Lucas' answer is correct

29 $30\sqrt{3} + 4\sqrt{5}$ units