Math 4SN St Thomas High School

# MIDTERM REVIEW 2019

NAME:

**TEACHER:** 

EXAM DATE & TIME: WEDNESDAY, DECEMBER 18<sup>TH</sup>, 2019 8AM – 11AM

Remember to bring (in a CLEAR ZIPLOC):

- Pencils, eraser, ruler, highlighters
- Calculator
- Memory aid:  $8\frac{1}{2}$  by 11 inch

SINGLE-SIDED

NAME in top right hand corner

Handwritten

## **BREAKDOWN OF EXAM**

The exam will consist of:

- 6 multiple choice (4 marks each)
- 4 short answer (4 marks each)
- 6 extended answer (10 marks each)

## Topics

## Algebra:

- Factoring
- Simplifying Radicals

#### Functions:

- Properties of Functions
- Linear Functions
- Quadratic Functions
- Greatest Integer Functions

#### Systems:

- Linear Systems (solving by elimination, substitution)
- Semi-Linear Systems
- Word Problems

#### Simplify.

1)  $\sqrt{18x^4}$ 2)  $\sqrt{512n}$ 

3) 
$$-3\sqrt{150u^3v^4}$$
 4)  $6\sqrt{112xy^2}$ 

5) 
$$\sqrt{15} \cdot \sqrt{6}$$
 6)  $-5\sqrt{15} \cdot \sqrt{6}$ 

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7) 
$$-5\sqrt{2} \cdot -3\sqrt{6}$$
 8)  $-3\sqrt{10} \cdot 5\sqrt{10}$ 

9) 
$$2\sqrt{5x^2} \cdot -\sqrt{5x^2}$$
 10)  $-\sqrt{10r^2} \cdot -4\sqrt{5r^3}$ 

11) 
$$-4\sqrt{3b}(\sqrt{6b} + \sqrt{5})$$
 12)  $\sqrt{3k}(\sqrt{10} + \sqrt{2})$ 

13) 
$$3\sqrt{6n}(4\sqrt{10n} + 5\sqrt{2n})$$
 14)  $4\sqrt{5k}(4\sqrt{10} - \sqrt{2})$ 

15) 
$$(5-5\sqrt{5})(-2-5\sqrt{5x})$$
  
16)  $(3\sqrt{3}-3\sqrt{5})(4\sqrt{3x}+2\sqrt{5})$ 

17) 
$$\frac{3}{3\sqrt{2}}$$
 18)  $\frac{\sqrt{4}}{2\sqrt{3}}$ 

19) 
$$\frac{\sqrt{2} - \sqrt{5}}{2\sqrt{10}}$$
 20)  $\frac{4 + 2\sqrt{3}}{\sqrt{2}}$ 

21) 
$$\frac{5}{-4-\sqrt{5}}$$
 22)  $\frac{5}{3-2\sqrt{2}}$ 

23) 
$$\frac{5}{3-3\sqrt{3x^4}}$$
 24)  $\frac{4}{-5k-5\sqrt{3k^3}}$ 

25) 
$$2\sqrt{2} + 2\sqrt{24} - 2\sqrt{54}$$
  
26)  $-2\sqrt{24} - 3\sqrt{24} + 3\sqrt{6}$ 

27) 
$$2\sqrt{2} - 3\sqrt{8} + 2\sqrt{2} - 3\sqrt{8}$$
 28)  $2\sqrt{54} - \sqrt{5} + 2\sqrt{24} + 2\sqrt{27}$ 

29) 
$$2\sqrt{3} - \sqrt{3} - 3\sqrt{6} + 2\sqrt{2}$$
 30)  $3\sqrt{5} - 2\sqrt{5} - \sqrt{45} - 2\sqrt{2}$ 



Find two binomials whose product is equivalent to  $3x^2 - 10x + 8$ .

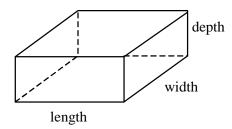


Factor the following polynomial :  $2x^2 - x - 15$ .

33

A company manufactured metal containers for their products.

The volume of these containers is given by the polynomial  $x(x^2 - 3x - 10)$ .



If x corresponds to the depth of a container, find the other two dimensions by factoring the trinomial  $x^2 - 3x - 10$ .

34 Factor each of the following polynomials.

- a)  $12xy^4 64x^3y^3 + 40x^2y^7$
- b)  $4x^2 + 28x 120$
- c)  $25b^2 36a^4$
- d) 6x + yx yz 6z

Factor the following polynomials:

a)  $y^2 - 1$ 

- b) x<sup>8</sup>-9
- c)  $16 25x^2$
- d)  $xyz x^3yz$
- e)  $x^2y 9y + ax^2 9a$
- f)  $(a + b)^2 16$
- g)  $(3x + y)^2 (x y)^2$

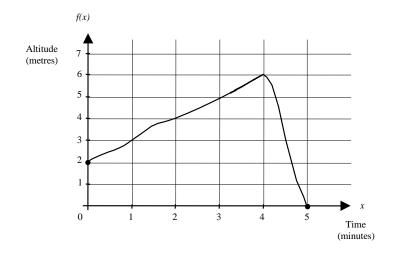
36

37

Factor the following polynomials:

- a) 18*x* + 20
- b) 12*x* + 4*y*
- c) 20xy + 16x
- d)  $m^8 + m^6$
- e)  $6x^2 2x + 4$
- f)  $a^2-ab-a$
- g)  $7x^5y^2 + 21x^2y^3 + 14xy^4$
- h) x(m+n) + y(m+n)
- i) c(a+b) (a+b)
- j)  $6x^2(3x-2) + 2x(3x-2) 4(3x-2)$

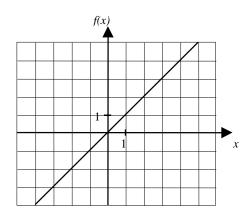
Given the function f(x) representing the altitude (in metres) of a bird as a function of the number of minutes from the time he left the branch of a tree to the moment he lands on the ground.



Which of the following statements is FALSE?

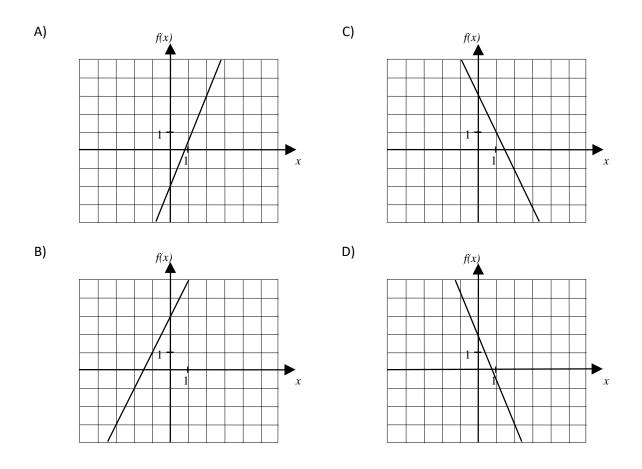
- A) f(x) is positive for all x in the interval [0, 5].
- B) f(x) is increasing for all x in the interval [0, 4].
- C) For f(x), the domain is [0, 5].
- D) For f(x), the range is [2, 6].

Andrea represented the function for which the rule is f(x) = x by the following graph :



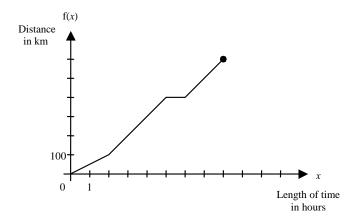
Then, she modified the parameters m and b to create function f(x) = mx + b

If, in her new rule, m = -2 and b = 3, which of the following graphs represents the new function?



Monica left home at noon (12 o'clock).

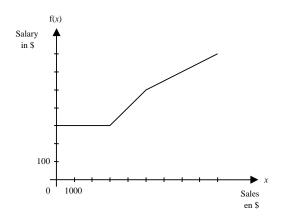
The following graph illustrates the distance in kilometres Monica travelled as a function of the length of time in hours since she left home.



What is the range of this function for the period from 2:00 p.m. to 6:00 p.m.?

A)	[0, 8]	C)	[2, 6]
B)	[0, 600]	D)	[100, 400]

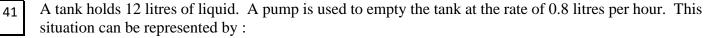
The following graph illustrates the weekly salary of a saleswoman as a function of the total value of her sales.



Within which interval does the saleswoman's salary fall if the total value of her sales is no more \$5000?

- A) [0, 500] C) [300, 700]
- B) [300, 500] D) [500, 700]

39



q(t) = 12 - 0.8t

t : time in hours that the pump is in action

q(*t*) : quantity of liquid left in the tank

How long will it take for the tank to be emptied?

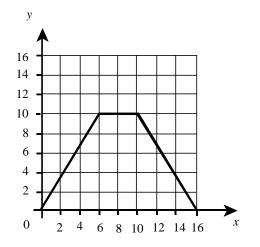
42

A) 12 hoursB) 15 hoursC) 9 hours 36 minutesD) 6 hours

The theoretical value V(x) of a car after a certain number of years in use is given by the following equation :

		$V(x) = 17\ 500 - 2500x$		
After how many years will the theoretical value be zero?				
A)	4 years	C)	6 years	
B)	5 years	D)	7 years	

43 The following Cartesian graph represents a function.



On which interval is this function increasing?

A) [0, 6]	C)	[6, 16]
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B) [0, 10] D) [10, 16]

The functions *f* and *g* are defined as follows :

$$f(x) = x^2 + 3x - 2$$
$$g(x) = 4x$$

What are the zeros of the function f - g?

- A) -2 and -1 C) -1 and 2
- B) -2 and 1 D) 1 and 2



Find the zeros of four 2<sup>nd</sup> degree functions.

- a)  $f(x) = 3x^2 x$
- b)  $f(x) = 15x^2 x 2$
- c)  $f(x) = 6(-3x^2 + 2x + 8)$
- d)  $f(x) = 2x^2 + x 3$



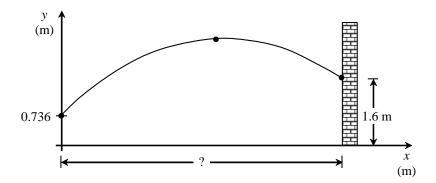
Evaluate each of the following for f(x) = 3x + 2

- a) f(0)
- b) f(-3)
- c) f(5)

Charles threw a ball toward a wall. The ball was at a height of 0.736 m when it left his hand.

The side view of the ball's trajectory is represented in the Cartesian plane below.

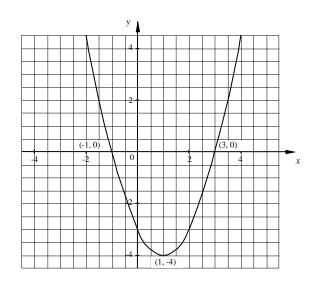
The rule  $f(x) = -0.1(x - 4.2)^2 + 2.5$  is associated with this trajectory. The scale of this graph is in metres.



When the ball hit the wall, it was at a height of 1.6 m and was falling to the ground.

How far away from the wall was Charles' hand when he threw the ball?

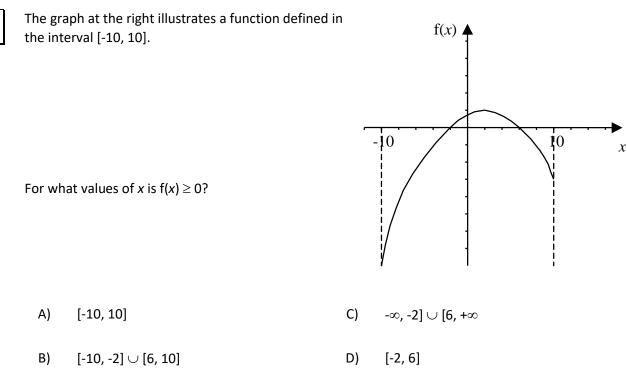
The parabola represented below crosses the x-axis at the points (-1, 0) and (3, 0) and its vertex is the point P(1, -4).



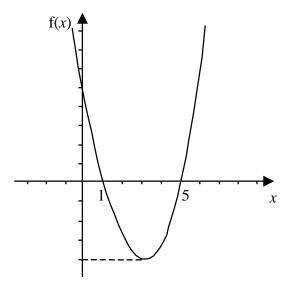
Among the following equations, which one represents the parabola graphed above?

A) 
$$y = 2x^2 + 2x - 3$$
 C)  $y = x^2 - 2x - 3$ 

B) 
$$y = x^2 + 2x - 3$$
 D)  $y = -x^2 - 2x + 3$ 



The parabola shown in the Cartesian plane intersects the *x*-axis at points (1, 0) and (5, 0) and the *y*-axis at (0, 5).



What is the rule of correspondence of this parabola?

A) 
$$f(x) = x^2 - x + 5$$
 C)

C)  $f(x) = x^2 - 6x + 5$ 

B)  $f(x) = -x^2 + 6x - 5$ 

D)  $f(x) = -x^2 + x + 5$ 

49

A mountain bike manufacturer employs 10 workers. The company's monthly production B(x) can be expressed by the following function :

$$B(x) = -x^2 + 60x + 1000$$

where B(x) represents the number of mountain bikes produced in one month and x represents the average number of years of experience of the workers in the company.

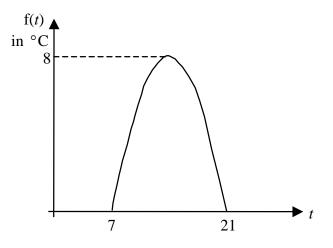
What is the maximum number of mountain bikes that this manufacturer can hope to produce in a month?

A)	30 bikes	C)	1000 bikes
B)	100 bikes	D)	1900 bikes

52

51

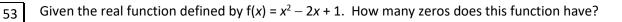
Temperature changes recorded on a given day in April can be represented by a quadratic function. The function is graphed below.



Which of the following rules of correspondence defines this function?

A) 
$$f(t) = -(t - 14)^2 + 8$$
 C)  $f(t) = -(t + 14)^2 + 8$ 

B) 
$$f(t) = \frac{-8}{49}(t - 14)^2 + 8$$
 D)  $f(t) = \frac{-8}{49}(t + 14)^2 + 8$ 



C)

Two

(-2, 2)

y

1

0

1

A) None

54

55

B) One D) An infinite number

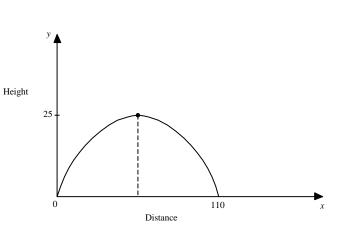
What function is represented by the parabola shown below?



The path of a ball hit during a baseball game is approximated by a parabola. This path is shown in the Cartesian plane below.

The ball reaches a maximum height of 25 m and hits the ground 110 m from home plate.

Which rule of correspondence defines the parabolic path travelled by the baseball?



(2, 2)

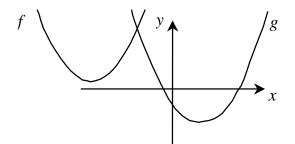
x

A) 
$$y = -x^2 + 110x$$
 C)  $y = \frac{x^2}{121} - \frac{110x}{121}$ 

B) 
$$y = \frac{-x^2}{121} + \frac{110x}{121}$$
 D)  $y = x^2 - 110x + 3050$ 

- The rule of correspondence of a real polynomial function of the second degree is  $f(x) = ax^2 + bx + c$ . If a > 0 and  $b^2 - 4ac < 0$ , which graph corresponds to this function?
  - A) A parabola that opens upward and that is tangent to *x*-axis.
  - B) A parabola that opens upward and that is located above the *x*-axis.
  - C) A parabola that opens downward and that is located below the *x*-axis.
  - D) A parabola that opens downward and that is tangent to the *x*-axis.

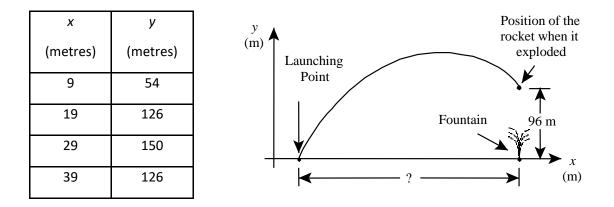
Functions *f* and *g* are represented by the parabolas shown below.



The rule of functions f and g are of the form  $y = a(x - h)^2 + k$ The graph of function g is obtained by transforming the graph of function f. Which of the following statements is true?

- A) The value of h decreases and the value of k decreases.
- B) The value of h decreases and the value of k increases.
- C) The value of h increases and the value of k decreases.
- D) The value of h increases and the value of k increases.

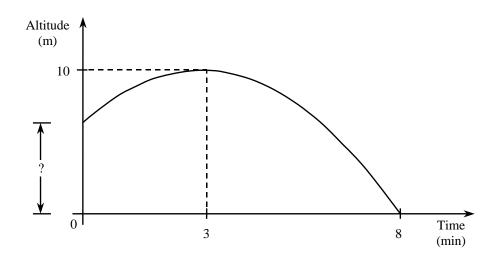
A rocket was launched during a fireworks show. The side view of the rocket's parabolic trajectory is represented by the following table of values and graph.



The rocket exploded 96 m above a fountain.

What is the distance between the point from which the rocket was launched and the fountain?

Melanie was playing with a remote-controlled toy airplane. The plane took off from a balcony and landed on the ground 8 minutes later. Three minutes after taking off, the plane reached a maximum altitude of 10 metres. In the graph below, the plane's altitude as a function of time is represented by a portion of a parabola.

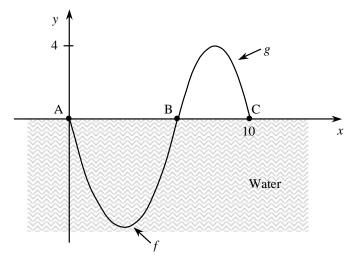


How high off the ground is the balcony located?

58

The following graph represents the side view of the path of a dolphin as it performs a trick during a show at an aquarium. This path is composed of portions of two parabolas associated with function *f* and *g* respectively.

The scale of the graph is in metres.



The rule  $f(x) = \frac{5}{9}(x-3)^2 - 5$  represents the dolphin's path when it is in the water.

When it is out of the water, the dolphin reaches a maximum height of 4 metres. The distance between points A and C is 10 metres.

What is the rule of the function g?

Caroline throws a ball toward a basket located 3 m above the ground.

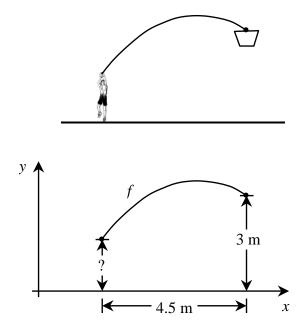
The ball reaches a maximum height. On its way down, it enters the basket.

In the Cartesian plane on the right, the side view of the ball's trajectory is represented by function *f*. The scale of this graph is in metres.

The rule associated with function *f* is

$$f(x) = -0.2(x-5)^2 + 3.45.$$

The horizontal distance between Caroline and the location of the basket is 4.5 m.



At the moment that Caroline throws the ball, what is the distance between the ball and the ground?

60

62 Caroline worked 50 hours last week. Her hourly salary is \$6 for regular working hours and \$9 for overtime. She earned a total of \$360. How many regular working hours did she work?

A)	20 hours	C)	41 hours
B)	30 hours	D)	44 hours

63 A clerk in a store sells programmable calculators for \$56 each and scientific calculators for \$23 each. At the end of a day, he had sold 64 calculators for a total of \$2132.

To determine the number (p) of programmable calculators and the number (s) of scientific calculators, the manager must use one of the following systems of equations. Which one?

A)	p + s = 64	C)	<i>p</i> + <i>s</i> = 2132
	23 <i>p</i> + 56 <i>s</i> = 2132		23 <i>p</i> + 56 <i>s</i> = 64
B)	<i>p</i> + <i>s</i> = 64	D)	<i>p</i> + <i>s</i> = 2132
	56 <i>p</i> + 23 <i>s</i> = 2132		56p + 23s = 64

Three customers are at the check-out of a local grocery store. The first pays \$4 for 2 litres of milk and 3 muffins. The second pays \$6.15 for 5 litres of milk and 2 muffins. The third has 1 litre of milk and 4 muffins.

How much will the third customer have to pay for his purchases?

64

A)	\$1.65	C)	\$3.75
B)	\$2.15	D)	\$4.50

For 3 compact disks and 4 cassettes it costs \$135. The cost of 2 compact disks is \$10 less than the cost of 4 cassettes.
What is the cost of 1 compact disk and 3 cassettes?

- A) \$90 C) \$70
- B) \$80 D) \$60

To go to school, Frank travels 1.8 km farther than Melanie. Twice the distance travelled by Frank equals 5 times the distance travelled by Melanie. Frank travels this distance 10 times per week.

What total distance does Frank travel in one week?

- A) 12 km C) 30 km
- B) 18 km D) 48 km

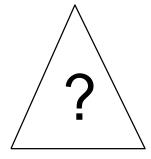
Solve the following systems of equations.

67

68

- a) x + 2y = 11 and x 3y = -9
- b) 2x + y = 7 and 4x 2y = 2

You want to construct a street sign in the shape of an isosceles triangle like the one shown below.



The sign must have the following properties:

- its perimeter must be 200 cm;

- Three times the length of the base must equal the sum of the two congruent sides.

In order to find the lengths of the sides of this sign, you begin to write a system of equations:

x : length (in cm) of each congruent side

y : length (in cm) of the base

1st equation : 2x + y = 200

2nd equation : \_\_\_\_\_

Write the missing equation.

To finance the year-end party, the graduating students sold agendas which brought in \$3.50 in profit per book.

In addition, they sold all of the pens that the supplier gave them on the basis of one pen for every 3 agendas ordered. Each pen gave them \$1.50 in profit.

How many agendas did they sell if they accumulated \$3000 in profit?

70

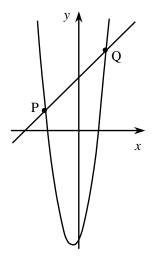
71

In the Cartesian plane on the right, points P and Q are the points of intersection of a line and a parabola.

The equation of the line is x - y + 11 = 0.

The equation of the parabola is  $y = x^2 + 2x - 19$ .

What are the coordinates of points P and Q?

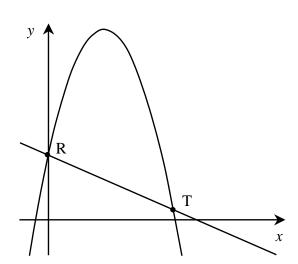


The equation of the parabola drawn in the Cartesian plane on the right is

 $y = -x^2 + 20x + 50.$ 

The equation of line RT is y = -2x + 50.

The parabola and the line intersect at points R and T.



Point R is located on the y-axis.

What are the coordinates of point T?

72 There are 120 tennis balls in a bag : some are white, some are green and some are yellow. The number of white balls equals the number of yellow balls. If there were 20 more green balls, the number of green balls would be double the number of white balls.

How many balls of each colour are there in the bag?

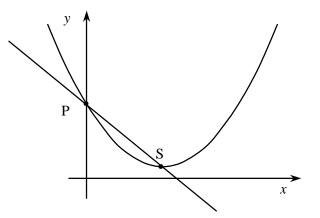
Annie and Mark each borrowed some money interest free. Annie borrowed \$500 which she will repay at a rate of \$40 per month. Mark borrowed \$600 which he will repay at a rate of \$60 per month. They both make their first payment at the same time.

After how many months will Annie's debt be equal to Mark's?

74

The equation of the parabola shown below is  $y = 4x^2 - 40x + 101$ .

Point S is the vertex of this parabola. The parabola intersects the *y*-axis at point P.



What is the equation of the line passing through points P and S?



76

In the Cartesian plane on the right, a straight line and a parabola intersect at points M and N.

The equation of the parabola is  $y = -2x^2 + 12x - 8$ .

Point M is the vertex of the parabola.

The *y*-intercept of the line is 22.

What are the coordinates of point N?

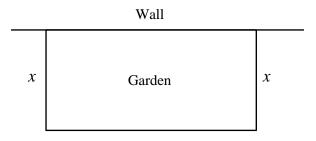
Terry wants to create a protected rectangular garden next to the house by erecting a fence on three sides of the plot as illustrated in the diagram below.

v

Μ

Ν

х



x = length of the congruent sides of the fence

Terry has 25 m of fencing and would like to enclose an area of at least 50 m<sup>2</sup>.

What is the smallest value of dimension *x* that represents this situation?

A stock with a market value of \$2 was purchased. An analysis shows that, for the 8-month period following the purchase, the value of the stock, V(t), varied according to the rule

$$V(t) = \frac{-1}{4}t^2 + 2t + 2$$

where t represents the number of months elapsed since the date of purchase.

For how many months was the value of the stock at least \$5?

A)	2 months	C)	6 months
B)	4 months	D)	8 months

78 An object is thrown into the air. The height it reaches is a function of the time elapsed expressed by the formula :

$$h = -10x^2 + 60x$$

Where *h* is the height in metres

and x is the time in seconds

In which interval of time will the projectile be located at a height more than 80 m?

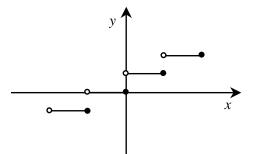
From the ground, a projectile is launched straight up at high speed. In meters, the height h(x) of the projectile is given by the following equation :

where x represents the number of seconds elapsed since lift off.

After lift off, how many seconds must elapse before the projectile reaches a height greater than 800 meters?

77

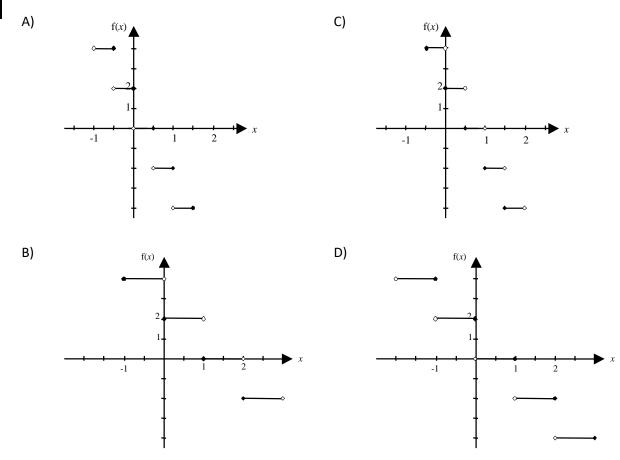
80 The rule of function *f* represented in the Cartesian plane below is f(x) = r[sx].



Which of the following statements is true?

A)	r > 0 and s > 0	C)	r < 0 and s > 0
B)	r > 0 and s < 0	D)	r < 0 and s < 0

81 Which of graphs below represents the equation f(x) = 2[-2x] + 2?



How much will it cost Danielle to send a parcel that weighs 4.4 kg?

It will cost \$\_\_\_\_\_\_ to send the parcel.

The weekly salary s(n) of a car salesperson is established by the equation

$$s(n) = 200 \left[ \frac{1}{2}(n+3) \right] + 200$$

where n is the number of cars sold in a week.

83

84

What salaries are possible for someone who sells fewer than 8 cars?

- A) \$500, \$600, \$700, \$800, \$900, \$1000, \$1100, \$1200
- B) \$0, \$600, \$800, \$1000, \$1200
- C) \$600, \$800, \$1000, \$1200
- D) \$400, \$600, \$800, \$1000, \$1200

A greatest integer function is defined by:

$$f(x) = 2[3x - 5] + 1$$

What is the range of this function?

A) 
$$\{y \in \Re \mid y = 2n, n \in Z\}$$
 C)  $\{y \in \Re \mid y = n + 1, n \in Z\}$ 

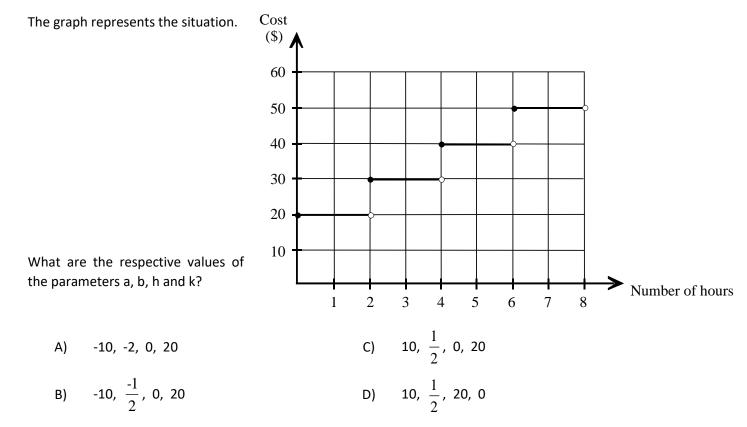
B) 
$$\{y \in \mathfrak{R} \mid y = 2n + 1, n \in \mathbb{Z}\}$$
 D)  $\{y \in \mathfrak{R} \mid y = 3n - 5, n \in \mathbb{Z}\}$ 

The rental cost C(x) of a specialized tool is given by an equation of the form

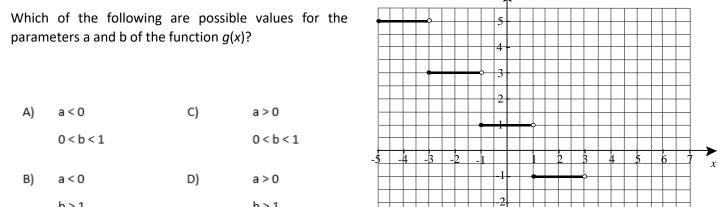
$$C(x) = a[b(x - h)] + k$$

where *x* represents the number of rental hours.

85



The graph of the function g(x) = a[bx - 1] + k is represented below. 86



g(x)

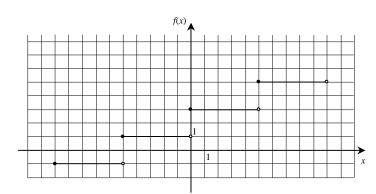
3

b>1 b>1 The function *f* is defined by the following rule:  $f(x) = 3\left[-\frac{(x-1)}{2}\right] + 6$ 

What are the zeros of this function?

A)	]1.5, 2[	C)	]5, 8[
B)	]3, 5]	D)	[5, 8[

Given the standard form of the greatest integer function: f(x) = a[b(x - h)] + k and the following graph.



What are the respective values of parameters a and b?

A) 
$$\frac{1}{2}$$
 and  $\frac{1}{5}$  C) 2 and  $\frac{1}{5}$ 

B) 
$$\frac{1}{2}$$
 and 5 D) 2 and 5

The basic greatest integer function, f(x) = [x], has been transformed into g(x) = a[b(x - h)] + k. Which of the following statements concerning the role of the parameters is false?

A) Parameter *a* affects the distance between the steps.

- B) Parameter *b* affects the length of the steps.
- C) Parameter *h* and *k* represent a horizontal and a vertical translation respectively.
- D) If parameter a < 0, and parameter b < 0, then function g(x) decreases over  $\Re$ .

89

87

A pastry chef orders sugar from his supplier. The cost of delivery, C(n), depends on the number, n, of kilograms of sugar ordered. The supplier charges a flat rate of \$10 for delivery. However, he gives a rebate of \$0.40 for every 100 kg of sugar delivered.

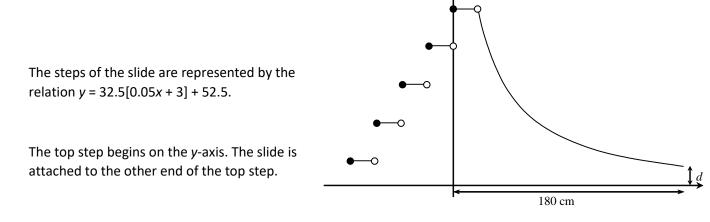
Quantity <i>n</i> of sugar	Cost of Delivery C(n)
ordered (kg)	(\$)
50	10
75	10
100	9.60
210	9.20
280	9.20

The pastry chef recorded the delivery costs for the last five orders in the table below.

The delivery costs for today's order was \$4.

What are all the possible quantities of sugar the pastry chef could have ordered today?

A designer is preparing a model of a children's slide. She began by drawing the steps and the slide on a Cartesian plane scaled in cm, as shown in the diagram below.



The slide is represented by a rational function with the equation  $y = a(x - 200)^2 + 30$ .

The end of the slide is 180 cm from the origin of the Cartesian plane.

To the nearest tenth of a centimetre, what is the distance (d) from the ground to the end of the slide?

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