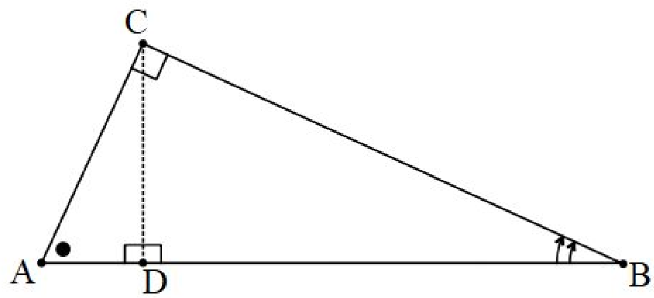
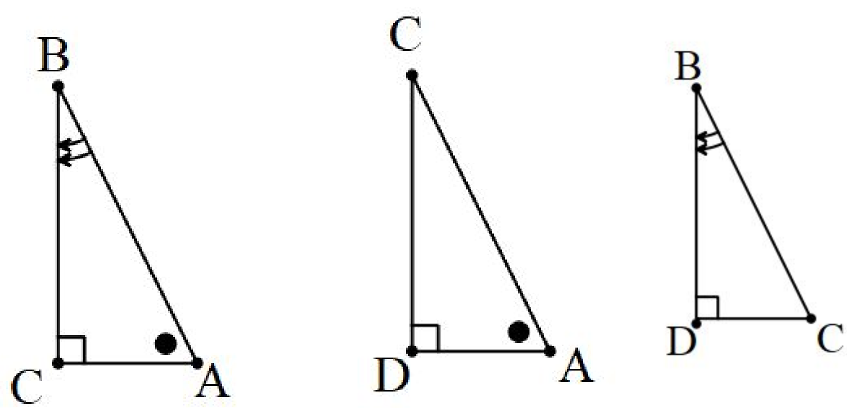
Metric Relations in Right Triangles

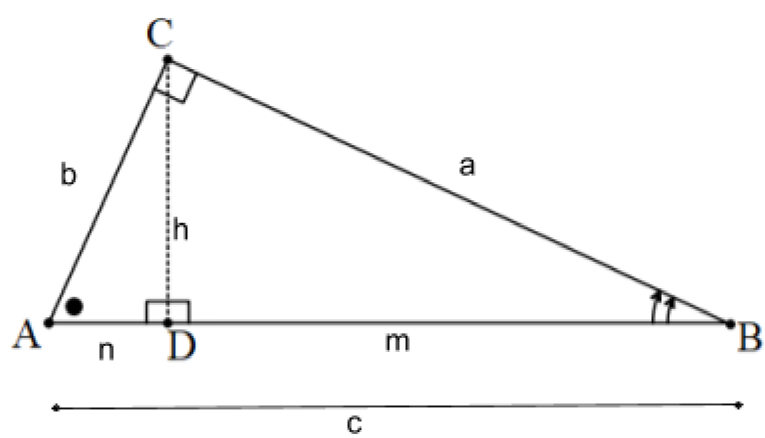
If we draw a height to the hypotenuse of a right triangle (shown below)



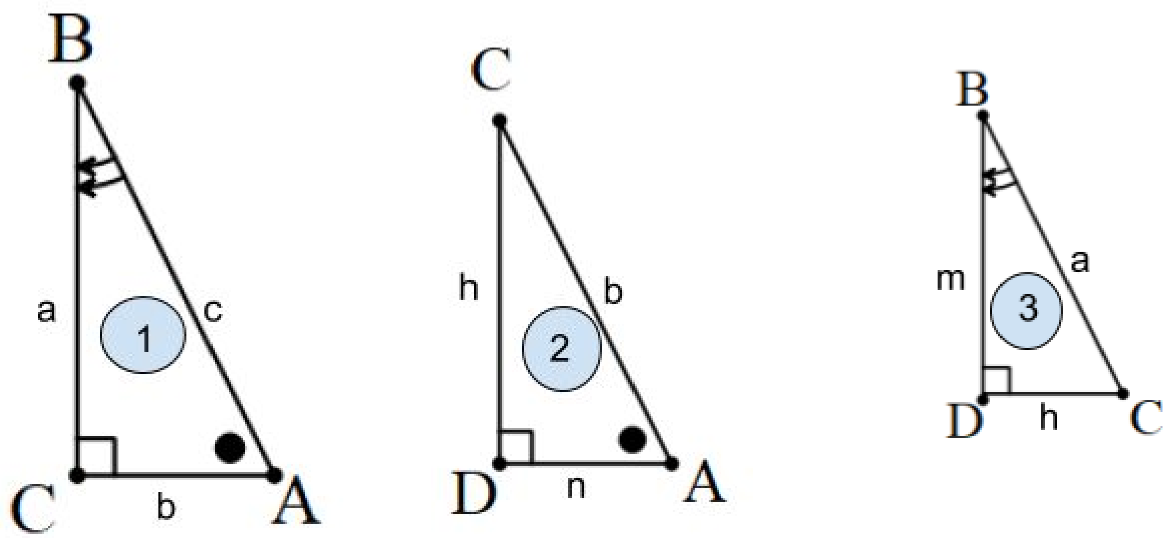
Whenever we do this, we create 3 similar triangles by AA (angle-angle) - these are drawn in the same orientation below:



We often use lower case letters to represent side lengths to simplify notation, so in the original drawing we will label sides as follows:



Which means in our 3 similar triangles we get



Knowing that similar triangles have proportional sides, we can write each of the following proportions:

- With triangles 1 and 2: $\frac{a}{h} = \frac{b}{n} = \frac{c}{b}$

    cross multiplying $\frac{b}{n} = \frac{c}{b}$ we get $b^2 = nc$ and

    cross multiplying $\frac{a}{h} = \frac{c}{b}$ we get $ab = hc$

- With triangles 2 and 3: $\frac{h}{m} = \frac{n}{h} = \frac{b}{a}$

    cross multiplying $\frac{h}{m} = \frac{n}{h}$ we get $h^2 = mn$

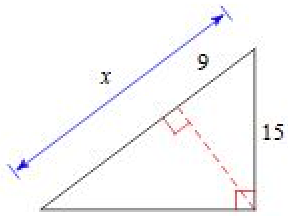
- With triangles 1 and 3: $\frac{a}{m} = \frac{b}{h} = \frac{c}{a}$

    cross multiplying $\frac{a}{m} = \frac{c}{a}$ we get $a^2 = mc$

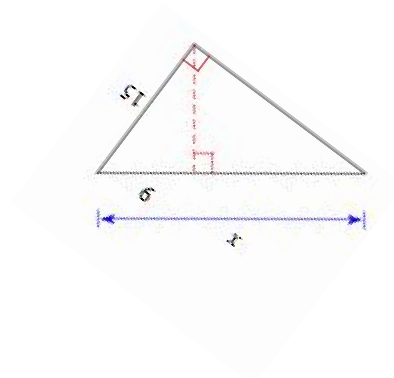
Using these four new equations, along with the Pythagorean Theorem, if you are given any two measurements, you can solve for all of the other measurements.

Example: In each of the following, solve for x.

1)



If we rotate the triangle as follows:



We see that b = 15, m = 9 and x represents c,

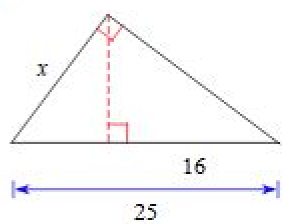
so we can substitute in $b^2 = nc$ to get

$15^2 = 9x$

$225 = 9x$

$25 = x$

2)



In this question b = x, m = 16 and c = 25

We can get n by subtracting n = 25 -16

n = 9

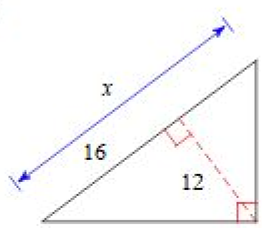
We can then use $b^2 = nc$ once again

$x^2 = 9\ (25)$

$x^2 = 225$

x = 15 (by square rooting both sides)

3)



Here we have h = 12, m = 16 and x = c,

We know that $h^2 = mn$

Therefore $12^2 = 16\ n$

$144 = 16\ n$

$9 = n$

This means x = 9 + 16

x = 25