Math 4CST – Systems Word Problems Practice

Part A: Substitution style questions.

1 The length of a rectangle is 8 cm more than its width. Its perimeter is 240 cm.

What is the area of this rectangle?

2 Edith bought some small tiles and some large tiles to recover a floor. She laid a total of 272 tiles. The design required 2.4 times as many large tiles as small ones.

At the end of the job, 40 small tiles and 16 large tiles were left over.

How many large tiles had she brought?

3 On the Jean-Lesage highway that joins Montreal to Quebec there is always a lot of traffic. One day last June, 27 200 vehicles were counted.

The cars outnumbered the trucks by 16 to 1.

Calculate the number of cars that were in circulation on the Jean-Lesage highway on that day.

4 In a particular neighbourhood, 3600 homes were robbed last year. A study showed that 4 times as many of these thefts were committed at night as during the daytime.

How many of these thefts were committed during the daytime?

5 Last Friday, 1200 people used the automatic teller at a bank in their neighbourhood.

There were 5 times as many people who made a withdrawal as those who made a deposit.

How many people made a deposit?

Part B: Elimination Style Questions

1 To pay for their team's travel expenses, the members of a football team buy caps and sweatshirts from a supplier and resell them for a profit.
The following table shows the number of items sold and the profit made by each of three team members.

	Number of Caps Sold	Number of Sweatshirts Sold	Profit
Eddie	8	20	$162
Gary	11	16	$148
Jonathan	6	?	$167

How many sweatshirts did Jonathan sell?

2 Nancy has hired a caterer to prepare a dinner to celebrate her grandparents' wedding anniversary.
The following are two bills for meals prepared by this caterer.

Bill	
Meals for	
15 adults	____
5 children	____
Total	$315

Bill	
Meals for	
10 adults	____
1 child	____
Total	$189

Nancy asks this caterer to prepare the same type of meal for 51 adults and 9 children.
What will be the total cost of the food for this dinner party?

3 A chemist wants to find out what chemical elements are used to make up a mixture. He knows that the mixture is made up of 2 of the following four elements..

Element	Atomic mass (g)
Aluminum	27
Calcium	40
Chromium	52
Copper	64

x and y represent the atomic masses of the unknown elements

The relationship between the sum and the difference of the atomic masses of the two unknown elements is given by
$$3x + 4y = 289$$
$$\text{and} \quad 3y - 5x = 21.$$

What elements make up the mixture?

| 4 | A school organizes a lottery to enable it to buy a computer. If each ticket sells for $2, the school will be short $1000. However,if they sell each ticket for $3, the school will make a profit of $250. How many tickets does the school need to print and what is the price of the computer? |

| 5 | At an amusement park, there is an entry fee and a charge for each ride on the merry-go-round. |

Three friends went to the park. Susan spent a total of $10.50 for the entry and 10 trips on the merry-go-round. Maria spent $13.50 to enter and take 14 rides on the merry-go-round.

How much did Sophie have to pay for the entry fee and 8 rides on the merry-go-round?

| 6 | Marcel took his family to a concert and paid $21.50 for 2 adult tickets and 5 children's tickets. |

He also paid $132 for 16 adult and 24 children's tickets for the members of his social club.

What is the value of the 2 adult tickets and 1 child's ticket which were used by the Smith family who are members of the club?

| 7 | A car rental company sets its prices on the basis of a fixed daily rate plus a certain amount per kilometre travelled. |

Two people rented cars. The first person travelled 300 km in one day and it cost her $105. The second person went 240 km in one day and it cost her $96.

What is the fixed daily rate charged by the company?

| 8 | In an automobile factory, the personnel department analyzed the causes of absenteeism among its employees. |

The situation is described by the following system of equations :

$$x = 240 - y$$
$$x = 4y + 40$$

where x represents the number of absences due to illness
and y represents the number of absences for personal reasons.

How many absences due to illness were recorded?

9 This year a gardener bought 3050 flower bulbs - tulips and lilies.

Because of increased demand, he decided to order twice as many lilies and three times as many tulips for next year. This will bring the total number of bulbs to 7950.

How many tulip bulbs will he order for next year?

10 A local high school ordered 20 balls for basketball and soccer for a total cost of $825.

The price of a basketball is $40 and a soccer ball is $45.

How many soccer balls did they buy?

11 For home repairs, a technician charges a fixed price plus a certain amount per hour of work. Therefore, it costs $145 for 3 hours of work and $100 for 1.5 hours of work.

How much would it cost for repairs that require 5 hours of work?

12 In a supermarket, a mix of 0.5 kg of peanuts and 0.75 kg of cashews costs $10.75. Likewise, a mix of 1 kg of peanuts and 1.25 kg of cashews costs $18.75.

According to this information, how much would a mix of 3.5 kg of peanuts and 2 kg of cashews cost?

13 To go to a rock concert with her friends, Joyce paid $214 for 4 balcony tickets and 3 main-floor tickets. To go to the same concert, Eric paid $208 for 5 balcony tickets and 2 main-floor tickets.

To go to this concert, how much will Kim pay for 2 main-floor tickets?

Answers

Part A:

1. Given x : length of the rectangle
   y : width of the rectangle

   System of equations representing the situation
   $$x - y = 8$$
   $$2x + 2y = 240$$

   Solution of the system of equations
   $$x = 64$$
   $$y = 56$$

   Area of the rectangle
   $$64 \times 56 = 3\ 584$$

   Result : The area of the rectangle is 3584 cm^2.

2. Given x, the number of large tiles used
   y, the number of small tiles used

   The system of equations :
   $$x + y = 272$$
   $$x = 2.4y$$
   Solution of the system of equations
   $$x = 192 \text{ and } y = 80$$
   The total number of large tiles bought
   $$192 + 16 = 208$$
   Result : She had bought 208 large tiles.

3. Let x : the number of cars
   y : the number of trucks
   System of equations
   $$x + y = 27\ 200$$
   $$x = 16y$$
   Solution : by substitution (or another method)
   $$x + y = 27\ 200$$
   $$16y + y = 27\ 200$$
   $$17y = 27\ 200$$
   $$y = 1600 \text{ (number of trucks)}$$
   Number of cars
   $$x = 16y$$
   $$x = 16 \times 1600$$
   $$x = 25\ 600$$ Result : The number of cars that circulated on the Jean-Lesage highway is 25 600.

4 Given x = the number of daytime thefts
and y = the number of nighttime thefts.

Since there were 3600 thefts in all,
$$x + y = 3600$$

Since there were 4 times as many nighttime thefts as daytime thefts
$$y = 4x \quad \text{or} \quad 4x - y = 0$$

The following system of equations represents this situation
$$x + y = 3600$$
$$4x - y = 0$$

Solve the system by reduction.

$$\begin{array}{rl} 1) & x + y = 3600 \\ +2) & \underline{4x - y = 0} \\ & 5x = 3600 \end{array}$$

therefore : $x = \dfrac{3600}{5} = 720$

There were 720 daytime thefts.

5 Given x = number of people who made a deposit
        y = number of people who made a withdrawal

"1200 people used the automatic teller ..." can be translated by
$$x + y = 1200$$

"5 times as many people who made a withdrawal as those who made a deposit" can be translated by
$$5x = y$$

Substitute $5x$ for y in the equation $x + y = 1200$.

This gives $x + 5x = 1200$; $6x = 1200$ and $x = 200$

Result : 200 people made a deposit.

Part B

1   System of equations representing the situation

x: profit per cap sold
y: profit per sweatshirt sold

$$8x + 20y = 162$$
$$11x + 16y = 148$$

Solving the system of equations

$$11(8x) + 11(20y) = 11(162)$$
$$8(11x) + 8(16y) = 8(148)$$

$$88x + 220y = 1782$$
$$\underline{88x + 128y = 1184}$$
$$92y = 598$$
$$y = 6.5$$

Since
$$y = 6.5$$
$$8x + 20(6.5) = 162$$
$$8x + 130 = 162$$
$$8x = 32$$
$$x = 4$$

Number of sweatshirts Jonathan sold
     Profit for the caps Jonathan sold: $6 \times \$4 = \$24$
     Profit for the sweatshirts Jonathan sold: $\$167 - \$24 = \$143$
     Number of sweatshirts Jonathan sold: $\$143 \div \$6.50 = 22$
Answer: Jonathan sold **22** sweatshirts.

2 x: the cost of a meal for an adult, in $
y: the cost of a meal for a child, in $

System of equations
$$15x + 5y = 315$$
$$10x + y = 189$$
Solving the system
$$15x + 5y = 315$$
$$5 \times (10x + y = 189)$$

$$\begin{array}{r} 15x + 5y = 315 \\ 50x + 5y = 945 \\ \hline -35x = -630 \\ x = 18 \end{array}$$

If $x = 18$ then $10 \times 18 + y = 189$
$$180 + y = 189$$
$$y = 9$$
Total cost of the food for the dinner party
> 51 meals for adults at $18 per meal: $918
> 9 meals for children at $9 per meal: $81

Total cost
$$\$918 + \$81 = \$999$$
Answer: The total cost of the food for this dinner party is $999.

3 To find the unknown elements, we must solve the system of equations by reduction.
$$3x + 4y = 289 \text{ (1)}$$
$$-5x + 3y = 21 \text{ (2)}$$
By multiplying equations (1) and (2) by 3, and by 4 respectively, we get an equivalent system.
$$9x + 12y = 867$$
$$-20x + 12y = 84$$
Subtract these 2 equations.

$$\begin{array}{r} 9x + 12y = 867 \\ -20x + 12y = 84 \\ \hline 29x = 783 \\ x = 27 \end{array}$$

The first unknown element has an atomic mass of 27 grams. It must be aluminum.

Since $x = 27$, then $3x + 4y = 289$ becomes
$$3(27) + 4y = 289$$
$$81 + 4y = 289$$
$$4y = 208$$
$$y = 52$$
The second element has an atomic mass of 52 grams. It must be chromium.

Result : The unknown elements are aluminum and chromium.

4 Given x, the number of tickets

   y, the price of the computer

System of equations

$$2x = y - 1\,000$$
$$3x = y + 250$$

Solution of the system of equations

$$x = 1250 \text{ and } y = 3500$$

Result : The school has to print <u>1250</u> tickets and the computer costs $<u>3500</u>.

5 System of equations x : entry fee

   y : cost of a ride on the merry-go-round

$$x + 10y = 10.50$$
$$x + 14y = 13.50$$

Solution of the system of equations

$$x = \$3.00$$
$$y = \$0.75$$

Determining the cost of entry and 8 rides on the merry-goround

   C : total cost

$$C = x + 8y = 3.00 + 8 \times 0.75 = 9.00$$
$$C = 9.00$$

Result : $9.00

6 Given x : the price of a child's ticket

   y : the price of an adult's ticket

$$5x + 2y = 21.50$$
$$24x + 16y = 132$$

Solving the system of equations

   $5x + 2y = 21.50$ The values of each type of ticket can be found.

   $8x + 4y = 33$

Use the method of reduction.

$$2(5x + 2y = 21{,}50) <\text{--->} \qquad\qquad 10x + 4y = 43$$
$$-\quad 6x + 4y\ = 33 \qquad\qquad\qquad - \underline{6x + 4y = 33}$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 4x = 10$$
$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = 2.50$$

If $x = 2.50$, then $5x + 2y = 21.50$ becomes

$$5(2.50) + 2y = 21.50$$
$$12.50 + 2y = 21.50$$
$$2y = 9$$
$$y = 4{,}50$$

therefore $2(4.50) + 2.50 = \$11.50$

7 Given x : fixed daily rate

y : additional amount per kilometre travelled.

The first person travelled 300 km and it cost her $105.

$$105 = x + 300y$$

The second person went 240 km and it cost her $96.

$$96 = x + 240y$$

Solve this system to find the fixed daily rate (x) charged by the car rental company.

$105 = x + 300y$ is an equation equivalent to $105 - 300y = x$

$96 = x + 240y$ is an equation equivalent to $96 - 240y = x$

By the method of comparison:

$$x = 105 - 300y = 96 - 240y = x$$
$$\text{therefore} \quad 105 - 300y = 96 - 240y$$
$$9 = 60y$$

and $y = \dfrac{9}{60} = 0.15$

If $y = 0.15$ then $x = 105 - 300(0.15) = \$60 =$ the fixed daily rate.

Result : The fixed daily rate is $60 .

8 Solve the given system of equations by comparison.
$$x = 240 - y = 4y + 40 = x$$
$$240 - y = 4y + 40$$
$$5y = 200$$
$$y = 40$$
If $y = 40$, $x = 240 - y = 240 - 40 = 200$

Result : 200 absences due to illness.

9 Given x : number of lily bulbs bought this year.
  y : number of tulip bulbs bought this year.

3050 flower bulbs : $x + y = 3050$
twice as many lily bulbs and three times as many tulip bulbs for a total of 7950 flower bulbs :
$$2x + 3y = 7950$$
Solve the system of equations by substitution.
$$x + y = 3050 \text{ or } x = 3050 - y$$
Replace x by $(3050 - y)$ in $2x + 3y = 7950$
$$2(3050 - y) + 3y = 7950$$
$$6100 - 2y + 3y = 7950$$
$$y = 1850$$
This year the gardener bought 1850 tulip bulbs. Next year he wants to buy three times as many. That is, $3 \times 1850 = 5550$ bulbs.

10 Given p : the number of basketballs
  s : the number of soccer balls

Purchase of 20 balls in all
$$p + s = 20 \text{ or } p = 20 - s$$

Cost of the 20 balls = $825
$$40p + 45s = 825$$

Since $p = 20 - s$, replace p in $40p + 45s = 825$
by $(20 - s)$

$$40p + 45s = 825 \text{ becomes}$$

$$40(20 - s) + 45s = 825$$
$$800 - 40s + 45s = 825$$
$$800 + 5s = 825$$
$$5s = 25$$
$$s = 5$$
Result : 5 soccer balls.

11 Equation that defines the situation

C : cost of the repairs
f : fixed price
x : amount per hour of work
h : number of hours of work

$C = hx + f$

Solving the system of equations

$145 = 3x + f$
$100 = 1.5x + f$
$f = 55$ and $x = 30$

Cost of repairs requiring 5 hours of work

$C = 5 \times 30 + 55$
$C = 205$

Result : The repairs would cost $205.

12

x : price of 1 kg of peanuts
y : price of 1 kg of cashews

$0.5x + 0.75y = 10.75$
$x + 1.25y = 18.75$

By solving the system, $x = 5$ and $y = 11$.
Therefore, 1 kg of peanuts costs $5 and 1 kg of cashews costs $11.

Price of the new mix

$(3.5 \times 5) + (2 \times 11) = \$39.50$

Result : The price of the new mix is $39.50.

13 Given x : price of one balcony ticket
y : price of one main-floor ticket

System of equations representing this situation

$4x + 3y = 214$
$5x + 2y = 208$

Solution of the system of equations

$x = 28$
$y = 34$

Price of Kim's tickets

$2 \times 34 = 68$

Result : Kim will pay $68 for 2 main-floor tickets.